The purpose of this phase of the project is to implement the core of the propositional resolution refutation theorem prover: the actual resolution engine. The project is to be implemented in SML. There are two principal tasks involved.

**Task 1**

You must write the function `resolve`, which, given two clauses (i.e. two lists of literals), returns a list of all the clauses that can result from any single application of the resolution rule. For example, if you give it the pair:

\[ [a, c, \neg d] \quad [b, \neg c, e] \]

It would produce the singleton list of clauses:

\[ [[a, b, \neg d, e]] \]

since they can only be resolved in one way. But if you gave it the pair:

\[ [a, \neg b, c, \neg d] \quad [b, \neg c, e] \]

It would produce the list of clauses:

\[ [[a, b, \neg d, e], [a, c, \neg c, \neg d, e]] \]

since there are two ways to resolve the pair of clauses. Thus the type of this function is `wff list -> wff list -> wff list list`.

**Task 2**

You must implement one main function: `consequence`, which, given a list of formulas and a single formula, proves whether the single formula is a logical consequence of the list of formulas. The type of this function is `wff list -> wff -> bool`.

To accomplish this, `consequence` should first convert each of the formulas in the list to CNF (by calling `cnf_list` from the first phase), and convert the negation of the single formula
to CNF ((by calling \texttt{cnf} from the first phase). It should then put all the resultant clauses in a single list and send them to the function \texttt{refute} which attempts to build a resolution refutation of the set of clauses. Thus the type of this function is \texttt{wff list list -> bool}.

This phase should be submitted using \texttt{cs80submit} as assignment 2, problem 1. You should submit just the functions \texttt{resolve, consequence, and refute}, and any support functions you write for them. While these functions call \texttt{cnf_list, and cnf} You should not include those functions. We will test your submission using the sample solutions for those functions. Your file also should not include (directly, or by use) the \texttt{wff} type definition.

**Extra Credit**

There are several extra-credit options:

- (15%) When refutation succeeds, return a data structure from which the refutation proof can be extracted.

- (15%) When refutation fails, return a satisfying valuation for the set of clauses.

- (5% each) Implement one of the (pre-)optimizations of resolution discussed in class.

You should note in the header of your submission for the project which, if any, extra-credit portions you are attempting.