Circularity

Problem: An ordering is a kind of relation, which is a subset of a cartesian product, which is a set of *ordered* pairs!

Solution: Do not appeal to ordered pairs. Create a new representation of pairs that does not require the notion of ordering. In a sense, we are providing a new *implementation* of the ordered pairs, which are now viewed as an *abstract data type*.

Redefining Ordered Pairs

Construction: First, we lift all values to singleton sets containing those values, by the obvious isomorphism. We then define the pairing notation as:

$$(x, y) \triangleq \{x, x \cup y\}$$

That is:

$$(\{x\}, \{y\}) \triangleq \{\{x\}, \{x, y\}\}$$

The destructors of the type are given by first defining the following operations:

$$I(S) \triangleq \bigcap_{A \in S} A$$

$$U(S) \triangleq \bigcup_{A \in S} A$$

Then the destructors are defined as:

$$fst(p) \triangleq I(p)$$

$$snd(p) \triangleq U(p) - I(p)$$