Circularity

**Problem:** An ordering is a kind of relation, which is a subset of a cartesian product, which is a set of *ordered* pairs!

**Solution:** Do not appeal to ordered pairs. Create a new representation of pairs that does not require the notion of ordering. In a sense, we are providing a new *implementation* of the ordered pairs, which are now viewed as an *abstract data type.*
Redefining Ordered Pairs

**Construction:** First, we lift all values to singleton sets containing those values, by the obvious isomorphism. We then define the pairing notation as:

\[(x, y) \triangleq \{x, x \cup y\}\]

That is:

\[(\{x\}, \{y\}) \triangleq \{\{x\}, \{x, y\}\}\]

The destructors of the type are given by first defining the following operations:

\[I(S) \triangleq \bigcap_{A \in S} A\]

\[U(S) \triangleq \bigcup_{A \in S} A\]

Then the destructors are defined as:

\[\text{fst}(p) \triangleq I(p)\]

\[\text{snd}(p) \triangleq U(p) - I(p)\]