

## Some Intrinsically Classical Sequent Proofs

$$\frac{\vdots}{\longrightarrow \phi \vee \neg\phi}$$

$$\frac{\vdots}{\longrightarrow (a \Rightarrow b) \equiv (\neg b \Rightarrow \neg a)}$$

## Pierce’s Formula

Now is a good time to revisit Pierce’s Formula:

$$((p \Rightarrow q) \Rightarrow p) \Rightarrow p$$

The NK proof of the formula is not very edifying:

$$\frac{\vdots}{((p \Rightarrow q) \Rightarrow p) \Rightarrow p}$$

The Sequent Calculus (LK) proof is more informative:

$$\frac{\vdots}{\longrightarrow ((p \Rightarrow q) \Rightarrow p) \Rightarrow p}$$

## Sequent Calculus Rules

In addition to the other readings, the rules can also be read as specifying how to search for a falsifying valuation.

$$\overline{\Gamma, A \longrightarrow \Delta, A} \textit{id} \quad \overline{\Gamma \longrightarrow \Delta, \top} \top \quad \overline{\Gamma, \perp \longrightarrow \Delta} \perp$$

$$\frac{\Gamma, A, B \longrightarrow \Delta}{\Gamma, A \wedge B \longrightarrow \Delta} \wedge_L \quad \frac{\Gamma \longrightarrow \Delta, A \quad \Gamma \longrightarrow \Delta, B}{\Gamma \longrightarrow \Delta, A \wedge B} \wedge_R$$

$$\frac{\Gamma, A \longrightarrow \Delta \quad \Gamma, B \longrightarrow \Delta}{\Gamma, A \vee B \longrightarrow \Delta} \vee_L \quad \frac{\Gamma \longrightarrow \Delta, A, B}{\Gamma \longrightarrow \Delta, A \vee B} \vee_R$$

$$\frac{\Gamma \longrightarrow \Delta, A \quad \Gamma, B \longrightarrow \Delta}{\Gamma, A \Rightarrow B \longrightarrow \Delta} \Rightarrow_L \quad \frac{\Gamma, A \longrightarrow \Delta, B}{\Gamma \longrightarrow \Delta, A \Rightarrow B} \Rightarrow_R$$

$$\frac{\Gamma \longrightarrow \Delta, A}{\Gamma, \neg A \longrightarrow \Delta} \neg_L \quad \frac{\Gamma, A \longrightarrow \Delta}{\Gamma \longrightarrow \Delta, \neg A} \neg_R$$

$$\frac{\Gamma \longrightarrow \Delta, A, B \quad \Gamma, A, B \longrightarrow \Delta}{\Gamma, A \equiv B \longrightarrow \Delta} \equiv_L \quad \frac{\Gamma, A \longrightarrow \Delta, B \quad \Gamma, B \longrightarrow \Delta, A}{\Gamma \longrightarrow \Delta, A \equiv B} \equiv_R$$

## Some Sequent Calculus Proofs and Non-Proofs

$$\frac{\vdots}{\longrightarrow a \wedge b \Rightarrow a \vee b}$$

$$\frac{\vdots}{\longrightarrow a \vee b \Rightarrow a \wedge b}$$

$$\frac{\vdots}{\longrightarrow (a \Rightarrow b) \Rightarrow (b \Rightarrow a)}$$

$$\frac{\vdots}{\longrightarrow (a \Rightarrow b) \Rightarrow (\neg a \Rightarrow \neg b)}$$