Herbrand's Theorems

Lemma: Given a set of formulas, Γ, the set, Γ', of (existentially) closed formulas obtained by existentially quantiying all the free variables of Γ is satisfiable iff Γ is satisfiable.

Therefore, we may, WLOG, restrict our attention to closed formulas (sentences).

Further, in the context of closed sentences, there is no distinction between a formula being satisfiable in a particular interpretation, and its being valid in that interpretation.

Herbrand's Theorems

Theorem (3.9.6)**:** A set of closed clauses Γ has a model if and only if it has a Herbrand Model.

Theorem (3.9.7 – Skolem-Herbrand-Gödel)**:** If a set of clauses Γ is unsatisfiable, then some finite set of ground clauses formed from Γ is unsatisfiable.

Theorem (Herbrand)**:** A set of clauses Γ is provable if and only if some finite set of ground clauses formed from Γ is provable.