CS140: Algorithms

Spring 2001

Assignment 2

This assignment is due at the start of class on Thursday, Jan. 25. You may collaborate with your classmates to complete this assignment but you should reference your collaborators. You also must thoroughly understand any solutions you submit and must do your own write-up of those solutions. This assignment is worth 100 points (75 points if handwritten).

1. Algorithm $\mathcal{A}$ operates on inputs of size b^m for $m = 0, 1, \ldots$. On input of size 1 $\mathcal{A}$ makes c steps. On input of size $n > 1$ algorithm $\mathcal{A}$ makes a recursive calls, each on input of size n/b. Algorithm $\mathcal{A}$ then takes $f(n)$ steps to construct its solutions based on the solutions to the recursive calls. Write the recurrence relation for the running time of $\mathcal{A}$ for the following cases.

   (a) $a = 3, b = 4, f(n) = cn^{\frac{3}{2}}$

   (b) $a = 4, b = 3, f(n) = cn \log_b n$

2. Draw the work tree for $m = 0, 1, 2$ for algorithm $\mathcal{A}$, case (i). Answer the following questions about the work tree for input size $n = b^m$. Assume the root is at level 0.

   (a) What input size is associated with a node at level i.

   (b) What work is done at a node in level i.

   (c) How many nodes are at level i.

   (d) What is the total work done by the nodes at level i?

   (e) How many levels are there?

   (f) Write a finite series (summation) that represents the total work done by the algorithm on input size n.

3. Repeat the previous problem for algorithm $\mathcal{A}$, case ii.