CS140: Algorithms

Spring 2001

Assignment 10

This assignment is due by the start of class on Thursday, April 12. You may collaborate with your classmates to complete this assignment but you must reference your collaborators. In addition, you must do your own write-up. This assignment is worth 100 points (65 points if handwritten).

1. In the study of NP-completeness we focus on "decision problems," which differ from standard search problems in that we are only required to answer a yes/no question about the input. In practice, of course, we usually want to solve the search variations of these decision problems. In this exercise you'll show that the search variations of several decision problems are really no harder than the decision version. You'll do this by reducing the search problem to the decision problem. Your reductions should be polynomial-time.

   Dominating Set: A dominating set of a graph G is a subset of vertices W such that every vertex of G is either in W or is adjacent to a vertex in W.

   Input: A graph G and integer k
   Question: Does G have a dominating set of size at most k/

   Input: A graph G
   Search Output: A smallest dominating set of G

   Clique: A clique in a graph G is a subset of vertices W such that each pair of vertices in W is connected by an edge in G.

   Input: A graph G and an integer k
   Question: Does G have a clique of size at least k?

   Input: A graph G
   Search Output: A largest clique of G

   Hamiltonian Cycle: A Hamiltonian cycle of a graph G is a simple cycle that includes every vertex.

   Input: A graph G
   Question: Does G contain a Hamiltonian cycle?

   Input: A graph G
   Search Output: A Hamiltonian cycle of G if one exists and otherwise "No Hamiltonian cycle"

   3-coloring: A 3-coloring of graph G is an assignment of a unique color in {red, blue, green } to each vertex of G so that the endpoints of each edge are colored differently.

Input: A graph G
Question: Can G be 3-colored?

Input: A graph G
Search Output: A 3-coloring of G if one exists and otherwise "No 3-coloring"

3 Dimensional Matching: Let U, V, and W be disjoint sets such that $||U|| = ||V|| = ||W|| = q$. Let C be a subset of $U \times V \times W$; in other words C is a collection of triples (u, v, w), where $u \in U$, $v \in V$, $w \in W$. A 3 dimensional matching is a subset of C such that each element of U, V and W occurs in exactly one triple of C.

Input: Disjoint sets U, V, and W, where $||U|| = ||V|| = ||W|| = q$ and a subset C of $U \times V \times W$.
Question: Does C contain a three-dimensional matching?

Input: Disjoint sets U, V, and W, where $||U|| = ||V|| = ||W|| = q$ and a subset C of $U \times V \times W$.
Search Output: A three-dimensional matching from C, if one exists, and otherwise a statement that one does not exit.

2. For each decision problem above do the following:
   (a) Give a yes-instance.
   (b) Give a no-instance.
   (c) Show the problem is in NP.