CS140: Algorithms

Spring 2001

Assignment 11

This assignment is due by the start of class on Tuesday, April 17. You may collaborate with your classmates to complete this assignment but you must reference your collaborators. In addition, you must do your own write-up. This assignment is worth 100 points (65 points if handwritten).

1. (Trivial) The Largest Common Subgraph problem is defined as follows.

   Input: Graphs G_1 and G_2 and integer k,

   Question: Is there a graph G containing at least k edges that is a subgraph of both G_1 and G_2.

   (a) Give a Yes-instance of LCS.

   (b) Give a No-instance of LCS.

   (c) Cast this problem as a search problem. Show that the search problem is polynomial-time equivalent to the decision problem.

   (d) Show that LCS is in NP.

   (e) Show that LCS is NP-complete by giving a reduction from one of the NP-complete problems we discussed in class.

2. (Easy) Repeat the above exercise for the Minimum Sum of Squares problem defined below.

   Input: A set A of integers and two additional integers K and J.

   Question: Can the elements of A be partitioned into K disjoint sets $A_1, A_2, \ldots, A_K$ such that $\sum_{i=1}^{K}(\sum_{s\in A_i} s)^2 \leq J$?

3. Repeat the previous exercise for the Steiner Tree in Graphs problem defined below. (Hint: Reduce from Vertex Cover.)

   Input: A graph G, a subset R of the vertices of G, and an integer K.

   Question: Is there a subtree (not necessarily a spanning tree) of G that includes the vertices R and at most K edges?