CS140: Algorithms

Spring 2001

Solutions to Assignment 0

1. For each of the following functions $f(n)$ and $g(n)$ state the ranges of positive integers for which

   (i) $f(n) < g(n)$

   (ii) $f(n) = g(n)$

   (iii) $f(n) > g(n)$

   For example, if $f(n) = 2n$ and $g(n) = 2^n$ then (i) hold in the range $[3, \infty)$, (ii) holds in the range $[1, 2]$, and (iii) fails to hold for any positive integers.

   (a) $f(n) = 10n$ and $g(n) = 3^n$
   Ans. (i) $[1, 3]$, (ii) $[]$, (iii) $[4, \infty)$

   (b) $f(n) = n/10$ and $g(n) = 10 \lg n$ (i.e. $\log_2$)
   Ans. (i) $[1, 995]$, (ii) $[]$, (iii) $[996, \infty]$

   (c) $f(n) = sin(2\pi n)$ and $g(n) = cos(2\pi n)$
   Ans. $sin(2\pi n) = 0$ and $cos(2\pi n) = 1$ for all positive integers n so (i) $[1, \infty)$

   (d) In this case the functions are defined recursively as follows:

   $n = 1$**:** $f(n) = 1$, $g(n) = 1$

   $n > 1$ **and even:** $f(n) = f(n-1)$, $g(n) = 2^{f(n-1)}$

   $n > 1$ **and odd:** $f(n) = 2^{g(n-1)}$, $g(n) = g(n-1)$

   Ans. (i) $\{2, 4, 8, \ldots\}$, (ii) $[1]$, (iii) $\{3, 5, 7, \ldots\}$

   (e) $f(n) = \log_3 n^2$, $g(n) = 2 \lg n$
   Ans. (i) $[2, \infty]$, (ii) $[1]$, (iii) $[]$

2. Identify the following statements as true or false. You may assume that a, w, k and n are positive real numbers.

   (a) $w = a^{\log_a w}$
   Ans. True

   (b) $\log_a^k n = k \log_a n$
   Ans. False

   (c) $\log_a n^k = k \log_a n$
   Ans. True

   (d) $\log_a n = \log_a b \, * \, \log_b n$
   An. True

3. Give a function $f(n)$ that neither goes to infinity nor converges as n goes to infinity.

   Ans. $sin(\pi n)$