CS140: Algorithms

Spring 2001

Solutions to Assignment 2

1. Algorithm $\mathcal{A}$ operates on inputs of size b^m for $m = 0, 1, \ldots$. On input of size 1 $\mathcal{A}$ makes c steps. On input of size $n > 1$ algorithm $\mathcal{A}$ makes a recursive calls, each on input of size n/b. Algorithm $\mathcal{A}$ then takes $f(n)$ steps to construct its solutions based on the solutions to the recursive calls. Write the recurrence relation for the running time of $\mathcal{A}$ for the following cases.

   (a) $a = 3, b = 4, f(n) = cn^{\frac{3}{2}}$
   Ans. $T(n) = 3T(n/4) + cn^{\frac{3}{2}}$

   (b) $a = 4, b = 3, f(n) = cn \log_b n$
   Ans. $T(n) = 4T(n/3) + c \log_3 n$

2. Draw the work tree for $m = 0, 1, 2$ for algorithm $\mathcal{A}$, case (i).

   Answer the following questions about the work tree for input size $n = b^m$. Assume the root is at level 0.

   (a) What input size is associated with a node at level i. Ans. $n/4^i$

   (b) What work is done at a node in level i. Ans. $c(n/4^i)^{\frac{3}{2}} = cn^{\frac{3}{2}}/8^i$

   (c) How many nodes are at level i. Ans. 3^i

   (d) What is the total work done by the nodes at level i? Ans. $cn^{\frac{3}{2}}(3/8)^i$

   (e) How many levels are there? Ans. $m + 1$

   (f) Write a finite series (summation) that represents the total work done by the algorithm on input size n. Ans. $cn^{\frac{3}{2}} \sum_{i=0}^{m} (3/8)^i$

(g) Give a tight upper bound on your series. Ans. $O(n^{\frac{3}{2}})$.

3. Repeat the previous problem for algorithm $\mathcal{A}$, case ii.

(a) What input size is associated with a node at level i. Ans. $n/3^i$

(b) What work is done at a node in level i. Ans. When $i \neq m$ the work is $c \log_3(n/3^i) = c \log(3^{m-i}) = c(m-i)$. When $i = m$ the work is c.

(c) How many nodes are at level i. Ans. 4^i

(d) What is the total work done by the nodes at level i? Ans. When $i \neq m$ the total work is $c4^i(m-i)$. When $i = m$ the total work is $c4^m$.

(e) How many levels are there? Ans. $m+1$

(f) Write a finite series (summation) that represents the total work done by the algorithm on input size n. Ans. $c4^m + c\sum_{i=0}^{m-1}(m-i)$.

(g) Give a tight upper bound on your series. Ans. $O(n^{\log_3 4})$.