CS140: Exam 3 Name ______________________________
4/26/01

This exam consists of 6 problems and 14 pages. Before you begin the exam make sure your copy is complete. The exam is designed to be finished in under 3 hours but you may use up to 24 hours to complete the test. The test must be turned in by the end of the end of the day Sunday, April 29. You will receive 5 bonus points for turning in the exam at least 24 hours early.

Write your name above. Write the day and time that you start and stop the exam in the space provided.

Start Day/Time ______________________________

End Day/Time ______________________________

Your answers will be judged for clarity, conciseness, and correctness. The space provided should be adequate for your solutions. You should use some of the extra time allotted for the exam to write your answers carefully. Illegible, confusing, and/or incomplete ideas will be judged harshly.

You may use a one page cheat sheet but no other texts, notes, web pages, etc. during this exam. You may not discuss the exam with anyone.

The next page of the exam gives the definition of several NP-hard problems you may find useful.

Good luck!

Here are some NP-hard problems you may want to use in completing this exam.

1. Vertex Cover

   Input: A graph $G = (V, E)$ and an integer K

   Question: Is there a subset $W \subseteq V$ containing at most K vertices such that every edge of G is incident to some vertex in W?

2. 3-Colorability

   Input: A graph $G = (V, E)$

   Question: Can the vertices of G be colored *red, green,* and *blue* so that no adjacent vertices have the same color.

3. Hamiltonian Path

   Input: A graph $G = (V, E)$

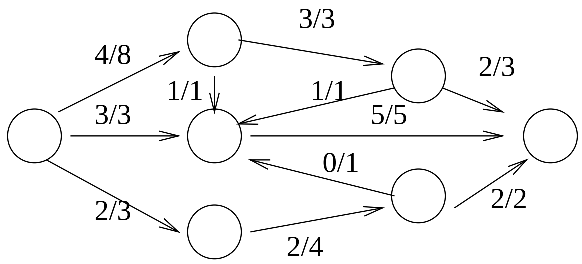
   Question: Does G contain a simple path that includes every vertex?

4. 3SAT

   Input: A boolean formula $\mathcal{F}$ in conjunctive normal form in which every clause contains exactly 3 literals.

   Question: Is $\mathcal{F}$ satisfiable?

1. Following is a flow network with a feasible flow. Note: The label X/Y on an edge indicates that the flow is X and the capacity is Y.



(a) Draw the residual graph.

(b) Find an augmenting path in the network. List the vertices of the path in order below.

(c) Augment the flow along this path as much as possible. Draw the network with the augmented flow below.

2. Construct a weighted graph G with distinct edge weights and containing a vertex s such that the Minimum Spanning Tree of G is not a Shortest Path Tree for s. Draw both the Minimum Spanning Tree and a Shortest Path Tree for S.

3. I've run the DFS algorithm on a directed graph G and constructed the following timestamp table.

Vertex	First Arrival	Last Departure
a	1	6
b	2	5
c	3	4
d	7	10
e	8	9

For each of the following statements, state whether the timestamp table implies that the statement is necessarily true, necessarily false, or if neither implication is made.

(a) Vertices a and c are strongly connected.

(b) Vertices a and d are strongly connected.

(c) Vertex a is reachable from vertex d.

(d) Vertex d is reachable from vertex a.

4. The Isomorphic Subtree problem is defined as follows.

Input: A graph G and a tree T.

Question: Is there a subset W of vertices of G such that the subgraph of G induced by W is exactly T?

(a) Show that the Isomorphic Subtree problem is in NP.

(b) Give a yes-instance of the problem.

(c) Give a no-instance of the problem.

(d) Show that the Isomorphic Subtree problem is NP-hard by giving a polynomial-time reduction from one of the NP-hard problems described in the instructions of this exam.

i. Which NP-hard problem will you use?

ii. Given an input X of the selected problem, how do you construct the input G, T for the Isomorphic Subtree problem? Show that the reduction can be done in polynomial-time.

iii. Suppose the input X of the selected problem is a yes-instance. Prove that the instance G, T of Isomorphic Subtree created by your reduction is also a yes-instance.

iv. Suppose that the instance G, T of Isomorphic Subtree created by your reduction is a yes-instance. Prove that the original instance X of the selected problem is also a yes-instance.

5. The Bipartite Cover Problem is defined as follows.

Input: A bipartite graph $G = (U, V, E)$ and an integer K.

Question: Is there a subset $W \subseteq U$ such that $||W|| \leq K$ and every vertex in V is adjacent to some vertex in W?

(a) Show that the Bipartite Cover problem is in NP.

(b) A search version of this problem takes as input a bipartite graph $G = (U, V)$ and outputs a smallest subset $W \subset U$ such that every vertex in V is adjacent to some vertex in U. Give a polynomial-time reduction from this search problem to the decision problem.

(c) Show that the Bipartite Cover problem is NP-hard by giving a polynomial-time reduction from one of the NP-hard problems described in the instructions of this exam.

i. Which NP-hard problem will you use?

ii. Given an input X of the selected problem, how do you construct the input G, K for the Bipartite Cover problem? Show that the reduction can be done in polynomial-time.

iii. Suppose the input X of the selected problem is a yes-instance. Prove that the instance G, K of Bipartite Cover created by your reduction is also a yes-instance.

iv. Suppose that the instance G, T of Bipartite Cover created by your reduction is a yes-instance. Prove that the original instance X of the selected problem is also a yes-instance.

6. The Shortest Common Supersequence problem is defined as follows.

Input: A set of strings $S = \{s_1, s_2, \ldots, s_n\}$ over an alphabet Σ and an integer K.

Question: Is there a string of length at most K that contains each $s \in S$ as a subsequence?

For example *abcd* is a shortest string that contains *ac* and *bd* as subsequences. (Other strings work as well.)

(a) Find a shortest string that contains *aba*, *cb*, and *caa* as subsequences.

(b) Show that the Shortest Common Supersequence problem is in NP.

(c) Show that the Shortest Common Supersequence problem is NP-hard by giving a reduction from Vertex Cover.

i. Given an instance $G = (V, E), K$ of Vertex Cover, show how to create the instance S, K' of Shortest Common Supersequence. Hint: Let $\Sigma = V$; i.e. there is a unique symbol for each vertex of the graph.

ii. Suppose that G, K is a yes-instance of Vertex Cover. Prove that the instance S, K' of Shortest Common Supersequence created by your reduction is also a yes-instance.

iii. Suppose that the instance S, K' of Shortest Common Supersequence produced by your reduction is a yes-instance. Prove that the input G, K is a yes-instance of Vertex Cover.