Compilation Considerations for Parallel and Vector Architectures

A Few Sources

- Michael Wolfe, High-Performance Compilers for Parallel Computing, Addison-Wesley, 1996.
- Hans Zima, with Barbara Chapman, Superompilers for Parallel and Vector Computers, ACM Press, 1990.
- Thomas Bräunl, Parallel Programming, an Introduction, Prentice-Hall, 1993.

Bernstein's Conditions (1966)

- For a statement S:
  - IN(S) = set of variables, registers, or locations used by S
  - OUT(S) = set written to by S
- S_1; S_2 (sequence) is **equivalent to** $S_1 \| S_2$ (parallel) **provided** that
  - $OUT(S_1) \cap OUT(S_2) = \emptyset$
  - $OUT(S_1) \cap IN(S_2) = \emptyset$
  - $OUT(S_2) \cap IN(S_1) = \emptyset$

Data Dependence

- Expresses constraints on parallel execution, as derived from sequential execution semantics
- Types of Dependence (Kuck, Wolfe, et al.):
  - Flow dependence
  - Anti dependence
  - Output dependence

Flow Dependence

- A variable assigned to in one statement is used in a later one:

A = 5

B = A*A

Flow Dependence
(now using Fortran notation for assignment: no semicolons)

Anti Dependence

- A variable used in one statement is assigned to in a later one:

B = A*A

A = 5

Anti Dependence

Output Dependence

- A variable assigned to in one statement is later re-assigned to:

A = B*B

Output Dependence

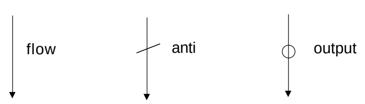
A = 5

Removable Dependences

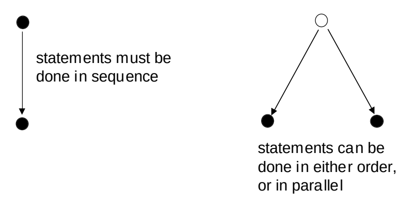
- Anti Dependence and Output Dependence are **removable.**
- They are **artifacts** of using **variables** as if memory **locations**, rather than purely for their **values**.
- Flow Dependence is **not removable** (unless the algorithm is changed); it expresses essential precedence.
- Clarification of whether **location-** or **value-**based dependency is being considered will be left to context.

Notation

- $S_1 \delta^{\mathbf{f}} S_2$ means S_2 is flow dependent on S_1
- $S_1 \delta^{\mathbf{a}} S_2$ means S_2 is anti dependent on S_1
- $S_1 \delta^{\mathbf{o}} S_2$ means S_2 is output dependent on S_1



Dependence Relations determine a Partial Order on Statement Execution



Location- vs. Value-Based

- Consider

Value-based Location-based

A = 5

B = A+7

A = 99

C = A*2

anti

output

By using a **different variable**, the dependency is removed

- Consider

Value-based Location-based

A = 5

B = A+7

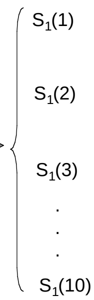
AA = 99

C = AA*2

Loops add to the Challenge

- Consider
  for K= 1 to 10
  $S_1(K)$ A[K] = B[K]

Dependency graph

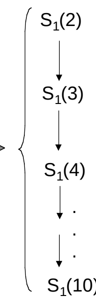


- Conclude: All instances $S_1(K)$ can be done **concurrently** (since no arrows).

Loops add to the Challenge

- Consider
  for K= 2 to 10
  $S_1(K)$ A[K] = A[K-1]

Dependency graph



- Conclude: All instances $S_1(K)$ must be done in sequence.

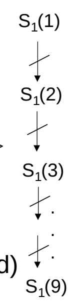
Larger offsets allow more concurrency

- Consider
  for K= 3 to 10
  $S_1(K)$ A[K] = A[K-2]
- $S_1(3)$ || $S_1(4)$ is possible
- $S_1(K)$ || $S_1(K+1)$ is possible, K = 3, 5, …
- Similarly, A[K] = A[K-d] will allow degree d concurrency.

"Forward" Offsets

- Consider
  for K= 1 to 9
  $S_1(K)$ A[K] = A[K+1]

Dependency graph

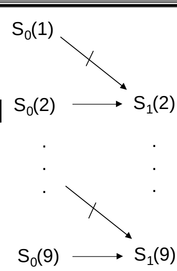


- Conclude: All instances $S_1(K)$ must be done in sequence (if **location-based** assumption used)

We can *Transform* the previous example

for K= 1 to 9
$S_0(K)$ B[K] = A[K+1]

for K= 1 to 9
$S_1(K)$ A[K] = B[K]

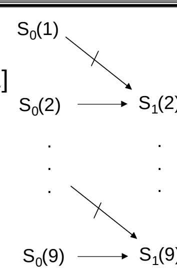


Transformation reduces sequence constraints

for K= 1 to 9
$S_0(K)$ B[K] = A[K+1]

for K= 1 to 9
$S_1(K)$ A[K] = B[K]

F90 style:
B(1 : 9) = A(2 : 10)
A(2 : 10) = B(2 : 10)



The type of transformation just shown can be automated

This is done routinely in compilers for high-performance machines.

Parallel Execution of Loops Strategy

- Try to issue different instances of a **loop body** to separate processing elements.
- Generally loops occur nested; try to find appropriate nesting level where different instances can be issued.

Similar issue to Parallelization: Vectorization

- Vector machines:
  - Exploit fine-grain parallel operations (+, *, /) on *vector* elements
  - Typically done with *vector registers*
- Vectorizing concentrates on **inner** loop
- Parallelizing concentrates on **outer** loops (coarser grain)

Example of Loop Vectorization

- for K = 1 to N
  A[K] = B[K] + C[K]
  D[K] = A[K]*5

uses new A

Vectorizes to (using F90 notation):

- A(1:N) = B(1:N) + C(1:N)
  D(1:N) = A(1:N)*5

uses new A

Example of Loop Vectorization

- for K = 1 to N
  A[K] = B[K] + C[K]
  D[K] = A[K+1]*5

uses old A

Vectorizes to (using F90 notation):

- D(1:N) = A(2:N+1)*5
  A(1:N) = B(1:N) + C(1:N)

uses old A

Note order reversal!

Mnemonic Aids

- for K = 1 to N
  A[K] = ...
  ... = A[K**+** d] $d > 0 \Rightarrow$ **old** value used
  $d \le 0 \Rightarrow$ **new** value used
  since assignment order is A[1], A[2], …
  but use order is A[1+d], A[2+d], ...
- for K = N to 1 by -1
  A[K] = ...
  ... = A[K **-** d] $d > 0 \Rightarrow$ **old** value used
  $d \le 0 \Rightarrow$ **new** value used
  since assignment order is A[N], A[N-1], …
  but use order is A[N-d], A[N-1-d], ...

Dependence Distance

- Extending our notation of statement dependence:

  $S_0\ \delta^{x}_{(d)}\ S_1$

  where $x \in$ {f, a, o}

  where d is a signed integer

  says:
  - For each i, S_0(i) must be done before S_1(i +d).

Dependence Distance

- for K = 2 to N-1

  S_0 A[K] = B[K]

  S_1 C[K] = A[K-1] $\delta^f_{(1)}$

  For each i, S_0(i) must be done before S_1(i + 1).

Both Indices and Loop Direction must be taken into account in determining Dependence Distance

- for K = 2 to N-1

  S_0 A[K] = B[K]

  S_1 C[K] = A[K-1] $\delta^f_{(1)}$

  is similar to

- for K = N-1 to 2 by -1

  S_0 A[K] = B[K]

  S_1 C[K] = A[K+1] $\delta^f_{(1)}$

same dependence distance: S_0(i) must be done before S_1 (i+1).

in that C[K] gets the *new* value, not the old.

Dependence Distance

- In general, there may be a different set of dependence distances for **each** array:

  for K = 2 to N

		for A	for B
S_0(K)	A[K] = B[K-1]	$\delta^f_{(0)}$	$\delta^f_{(1)}$
S_1(K)	B[K] = A[K]		

- Each places a constraint on loop restructuring

Direction Vectors

- Less precise than Dependence Distances, but frequently used:
  - $\delta^f_{(<)}$ used in place of $\delta^f_{(d)}$ where d > 0
  - $\delta^f_{(=)}$ used in place of $\delta^f_{(0)}$
  - $\delta^f_{(>)}$ used in place of $\delta^f_{(d)}$ where d < 0
- An advantage of using > is that d might not be fixed, as in:

  for K = 2 to 10

  A[2*K] = B[K]+1

  C[K] = A[K]
- Here the dependence distance *increases* with K.

Example

- for K = 2 to N

  S_0 A[K] = B[K]

  S_1 C[K] = A[K-1] $\delta^f_{(1)}$ or $\delta^f_{(<)}$

  for K = 1 to N-1

  S_0 A[K] = B[K]

  S_1 C[K] = A[K+1] $\delta^a_{(1)}$ or $\delta^a_{(<)}$

doacross (M. Wolfe)

- doacross K = M to N

is equivalent to HPF's INDEPENDENT annotation:

Each loop body is done independently of the others, possibly in parallel (There is still sequencing ***within*** the body.)

doacross Example

- Original loop

```
for K = 1 to N
    A[K] = C[K]
    B[K] = A[K]
```

$\delta^f_{(=)}$

- is optimized to

```
doacross K = 1 to N
    A[K] = C[K]
    B[K] = A[K]
```

Non-doacross Example

- Original loop

```
for K = 2 to N
    A[K] = C[K]
    B[K] = A[K-1]
```

$\delta^f_{(<)}$

cannot be optimized using doacross alone.

- We could provide additional synchronization on the use of A[K-1] to do it, but it wouldn't be pure doacross.

Loops that "Carry" Dependence

- As we saw, loops having only $\delta^f_{(=)}$ are optimizable using doacross.
- A loop with $\delta^f_{(<)}$ or $\delta^f_{(>)}$ **carries** a dependence that prevents parallel execution.

Nested Loops

- For nested loops, a *vector* of dependences is used, e.g. $\delta^f_{(=, <)}$ or $\delta^a_{(=, =)}$ with one component per loop nest.
- When loops are nested, the ***outermost*** loop with a $\delta^f_{(<)}$ or $\delta^f_{(>)}$ *carries* the dependence.

Nested Loop Example

- for K = 2 to N

```
for K = 2 to N
    for J = 2 to N
        A[K, J] = B[K, J]
        B[K, J] = A[K, J-1]
```

for A: $\delta^f_{(=, <)}$

for B: $\delta^a_{(=, =)}$

- The inner loop carries a dependence for A; no loop carries a dependence for B.
- Therefore the outer loop can be parallelized using doacross.

Nested Loop Example

- for K = 2 to N

```
for J = 2 to N
      A[K, J] = B[K, J]
      B[K, J] = A[K, J-1]
```

for A $\delta^f_{(=, <)}$ for B $\delta^a_{(=, =)}$

- doacross K = 2 to N

```
for J = 2 to N
      A[K, J] = B[K, J]
      B[K, J] = A[K, J-1]
```

Exercise

- How to parallelize:

```
for K = 2 to N
   for J = 2 to N
         A[K, J] = C[K, J]
         B[K, J] = A[K-1, J]
```

Exercise

- How to parallelize:

```
for K = 2 to N
   for J = 2 to N
         A[K, J] = C[K, J]
         B[K, J] = A[K-1, J]
```

for A $\delta^f_{(<, =)}$

- The outer loop carries a dependency, so cannot be parallelized.
- The inner loop can be parallelized.

Exercise

- for K = 2 to N

```
for J = 2 to N
      A[K, J] = C[K, J]
      B[K, J] = A[K-1, J]
```

for A $\delta^f_{(<, =)}$

- Parallel:

```
for K = 2 to N
      doacross J = 2 to N
            A[K, J] = C[K, J]
            B[K, J] = A[K-1, J]
```

Loop Interchanging

- for K = 1 to N

```
   for J = 2 to N
S1       A[K, J] = A[K, J-1] + A[K, J+1]
```

- $S_1\ \delta^f_{(=, <)}\ S_1$ implies inner loop cannot be vectorized.
- No dependencies of form $\delta^f_{(<, >)}$ implies loops can be interchanged

Loop Interchanging

- for K = 1 to N

```
for J = 2 to N
      A[K, J] = A[K, J-1] + A[K, J+1]
```

- for J = 2 to N

```
for K = 1 to N
      A[K, J] = A[K, J-1] + A[K, J+1]
```

- Now have $\delta^f_{(<, =)}$

Loop Interchanging

- for J = 2 to N
  for K = 1 to N
  A[K, J] = A[K, J-1] + A[K, J+1]
- Now have $\delta^f_{(<, =)}$
- Execute as:

  for J = 2 to N
  A[1:N, J] = A[1:N, J-1] + A[1:N, J+1]

Parallelization Rules

(as summarized by Thomas Bräunl)

- Data dependences '=' for the target loop do not have to be synchronized.
- Data dependences '<' or '>' in loops *outside* the target do not have to be considered.
- Loops *inside* the target do not have to be considered.
- All other data dependences need to be synchronized, such as with semaphores.
- Execution order inside the target loop may be changed, but '=' dependences must translate into enforced precedences.

Rule Examples

- Data dependence directions with **=** in the target loop do not have to be synchronized.

```
target  for i = 1 to n
S1   A[i] = C[i]
S2   B[i] = A[i]
```

$S_1 \, \delta^f_{(=)} \, S_2$

```
doacross i = 1 to n
S1   A[i] = C[i]
S2   B[i] = A[i]
```

Rule Examples

- Data dependences '<' or '>' in loops *outside* the target do not have to be considered.

```
        for i = 1 to n
target   for j = 1 to m
S1   A[i, j] = C[i, j]
S2   B[i, j] = A[i-1, j-1]
```

$S_1 \, \delta^f_{(<, <)} \, S_2$

```
for i = 1 to n
  doacross j = 1 to m
S1   A[i, j] = C[i, j]
S2   B[i, j] = A[i-1, j-1]
```

Rule Examples

- Loops *inside* the target do not have to be considered.

```
target  for i = 1 to n
         for j = 1 to m
S1   A[i, j] = B[i, j]
S2   B[i, j] = A[i, j-1]
```

$S_1 \, \delta^f_{(=, <)} \, S_2$

and

$S_1 \, \delta^f_{(=, =)} \, S_2$

```
doacross i = 1 to n
  for j = 1 to m
S1   A[i, j] = B[i, j]
S2   B[i, j] = A[i, j-1]
```

Gauss-Seidel & Wavefronts

- Outline
  - Gauss-Seidel vs. Jacobi
  - Wavefronts
  - Red-black
  - Chaotic approaches