Harvey Mudd College
Computer Science 80
Logic for Computer Science
Spring Semester 2001

Assignment #5 – Propositional Logic: Resolution Refutation and Matching
Sample Solution

1. Consider the set of numbered clauses:

$$\overset{1}{a \vee \neg b \vee c} \qquad \overset{2}{b \vee c \vee \neg d} \qquad \overset{3}{b \vee \neg c \vee e \vee \neg f} \qquad \overset{4}{c \vee d \vee g}$$

   For each pair of clauses show all possible resolvents of the pair.

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1 & 2: One Resolvent: $a \vee c \vee \neg d$

1 & 3: Two Resolvents: $a \vee c \vee \neg c \vee e \vee \neg f$ and $a \vee b \vee \neg b \vee e \vee \neg f$

1 & 4: No Resolvents

2 & 3: One Resolvent: $b \vee \neg d \vee e \vee \neg f$

2 & 4: One Resolvent: $b \vee c \vee g$

3 & 4: One Resolvent: $b \vee e \vee \neg f \vee d \vee g$

2. Consider the set of assumptions:

- Every fungus is either a mushroom or a toadstool.
- Every boletus is a fungus.
- Toadstools are poisonous, as are peach pits.
- A boletus is not a mushroom.
- This thing is a boletus.

If we wish to know whether "this thing" is poisonous, we can model this with the set of formulas:

$$\Gamma = \{f \Rightarrow (m \vee t), b \Rightarrow f, (pp \vee t) \Rightarrow p, b \Rightarrow \neg m, b\}$$

and attempt to prove the consequence:

$$\Gamma \models p$$

Construct the corresponding implication, convert its negation to conjunctive normal form, and produce a resolution refutation tree showing that the consequence holds.

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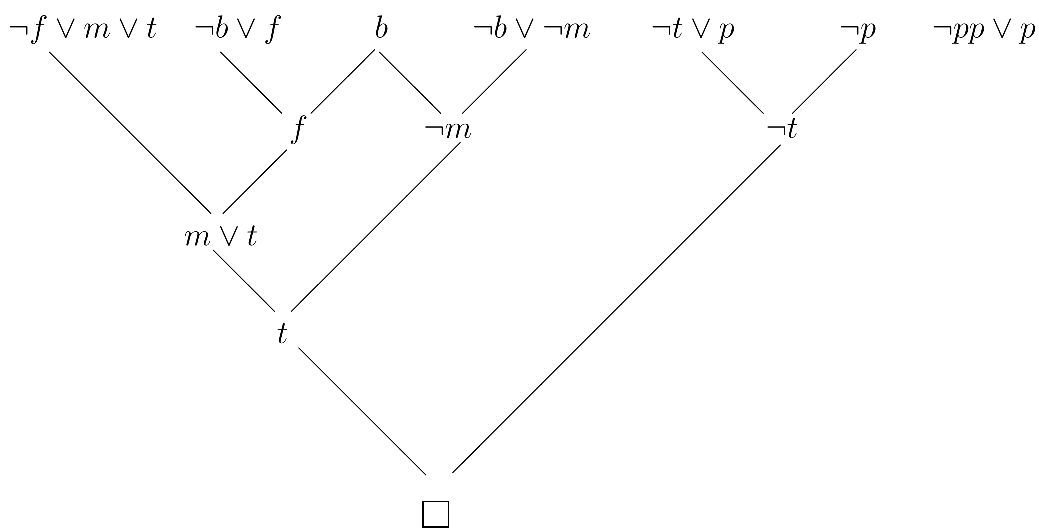
- Construct the corresponding implication and negate it:

$$\neg(((f \Rightarrow (m \vee t)) \wedge (b \Rightarrow f) \wedge ((pp \vee t) \Rightarrow p) \wedge (b \Rightarrow \neg m) \wedge b) \Rightarrow p)$$

- Convert it to CNF:

$$(\neg f \vee m \vee t) \wedge (\neg b \vee f) \wedge (\neg pp \vee p) \wedge (\neg t \vee p) \wedge (\neg b \vee \neg m) \wedge b \wedge \neg p$$

- Produce a resolution refutation for the set of clauses:



Note that there are many other resolvents possible at each level.

3. Consider the set of assumptions:

   - If the jar is heated, and the bug is in the jar, then the bug is dead.
   - The jar is heated.
   - The bug is in the jar.

   If we wish to know whether the bug is dead, we can model this with the set of formulas:

$$\Gamma = \{hj \Rightarrow (bij \Rightarrow bd), hj, bij\}$$

   and attempt to prove the consequence:

$$\Gamma \models bd$$

   Construct the corresponding implication, convert it (not it's negation) to CNF, and use the "simple" validity tester to show that the original consequence holds.

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   - Construct the corresponding implication:

$$((hj \Rightarrow (bij \Rightarrow bd)) \wedge hj \wedge bij) \Rightarrow bd$$

   - Convert it to CNF:
     - Convert implications using $(A \Rightarrow B) \leftrightarrow (\neg A \vee B)$:

$$\neg((\neg hj \vee \neg bij \vee bd) \wedge hj \wedge bij) \vee bd$$

     - Move negations inwards using DeMorgan duals and double negation cancelation:

$$(hj \wedge bij \wedge \neg bd) \vee \neg hj \vee \neg bij \vee bd$$

     - Distribute $\vee$ over $\wedge$:

$$(hj \vee \neg hj \vee \neg bij \vee bd) \wedge (bij \vee \neg hj \vee \neg bij \vee bd) \wedge (\neg bd \vee \neg hj \vee \neg bij \vee bd)$$

   - Identify clashing pairs *within* each clause:

$$(\underline{hj} \vee \underline{\neg hj} \vee \neg bij \vee bd) \wedge (\underline{bij} \vee \neg hj \vee \underline{\neg bij} \vee bd) \wedge (\underline{\neg bd} \vee \neg hj \vee \neg bij \vee \underline{bd})$$