

Harvey Mudd College  
Computer Science 80  
Logic for Computer Science  
Spring Semester 2001

Assignment #7 – Predicate Logic: Proof Theory  
Due 11:00am, Friday, April 20, 2001

1. Give Natural Deduction proofs of each of the following formulas (brackets are used in place of parentheses in some formulas to make the structure clearer):
  - (a)  $\exists x[\forall y(p(x, y))] \Rightarrow \forall y[\exists x(p(x, y))]$
  - (b)  $\exists x[p(x) \Rightarrow q(x)] \Rightarrow [\forall x(p(x)) \Rightarrow \exists x(q(x))]$
  - (c) Prove that any boletus is poisonous. That is, from open assumptions:
    - $\forall x(f(x) \Rightarrow [m(x) \vee t(x)])$
    - $\forall x(b(x) \Rightarrow f(x))$
    - $\forall x([t(x) \vee pp(x)] \Rightarrow p(x))$
    - $\forall x(b(x) \Rightarrow \neg m(x))$prove the conclusion  $\forall x(b(x) \Rightarrow p(x))$
  - (d) (extra credit)  $[\forall x(p(x)) \Rightarrow \exists x(q(x))] \Rightarrow \exists x[p(x) \Rightarrow q(x)]$
2. Give Gentzen Sequent Calculus proofs for each of the conclusions in the last problem, including the extra credit (which is not extra credit in this problem).
3. Give Resolution Refutation proofs for each of the conclusions in the first problem, including the extra credit (which is not extra credit in this problem).