The Principle of Complete Induction

Definition: Given a set A, a *property of* A is a function $P : A \to \{\bot, \top\}$. We say that *the property* P *holds for an element* $x \in A$ if $P(x) = \top$.

The Principle of Complete Induction:
Given a well-founded poset $(A, \leq)$, to prove that a property P holds for every element of A, it suffices to show that, for every element $x \in A$,

$$\text{if } P(y) \text{ holds for all } y < x \text{ then } P(x) \text{ holds}$$

This whole statement can be written notationally as:

$$(\forall x \in A)[(\forall y \in A)(y < x \Rightarrow P(y)) \Rightarrow P(x)] \Rightarrow (\forall z \in A)P(z)$$

The Principle of Complete Induction

Lemma: The Principle of Complete Induction holds for every well founded set.

Proof: