The purpose of this document is to give a concise summary of those features of Standard ML (a.k.a. SML) most useful for the first part of CS 131. The document is not intended as a first tutorial in functional programming, as it is assumed that you have some vague recollection of using rex in CS 60.

1 SML Philosophy

While imperative languages perform computation by creating variables and modifying their values, functional languages like SML are based on the idea of computation simply by evaluating definitions and expressions.

Every expression in SML:

• ...has a type such as int or bool or int*string->int.

• ...may result in a value (answer) when evaluated.

• ...may cause side-effects when evaluated.

A side-effect is anything that happens during evaluation besides computing values, such as assignments, input/output, or raising of exceptions. Not all functional languages allow side-effects; such languages are often called “pure” or “purely functional”. SML encourages purely functional programming, but also permits side-effects in those cases where they are appropriate.

In contrast to many functional languages including Lisp, Scheme, and rex, Standard ML is strongly typed. Programs are typechecked before they are run, and SML is very strict about its error-checking. This may seem annoying at first, but is actually extremely useful — error messages nearly always reflect a real bug in your code.

The notation

\textit{expression} : \textit{type}

is used to state the fact that the given SML expression has the specified SML type. For example,

\begin{verbatim}
  3 : int
\end{verbatim}

reflects the fact that in SML, the type of integers (including the constant 3) is written \texttt{int}. 

1
2 Numbers

There are two main numeric types in Standard ML, \texttt{int} and \texttt{real}. The type \texttt{int} classifies signed integers\(^1\) such as 42 and \texttt{~4}, while the type \texttt{real} classifies (double precision) floating-point numbers such as 16.0, \texttt{~14.237}, and \texttt{~1.2e~14}. Unary negation is written with a tilde; this distinguishes it syntactically from the binary subtraction operator.

The ordinary arithmetic operators work on pairs of integers or on pairs of reals, except that \texttt{div} is the operator for integer division (which truncates its result to another integer) while \texttt{/} is used for floating-point division:

\[
\begin{align*}
(3 + 4 \times (\texttt{~6} - 18 \texttt{ div} 4)) \texttt{ mod} 15 & : \texttt{int} \\
6.02\texttt{e}~23 + 4.5 \times (\texttt{~6.0} - 18.3 / 4) & : \texttt{real}
\end{align*}
\]

SML \textit{never} implicitly converts values from one type to a different type, so you cannot apply the arithmetic operators to one integer and one real; you have to first explicitly convert the integer to a real (via the function \texttt{real}) or else convert the real to an integer (via one of the functions \texttt{round}, \texttt{ceil}, or \texttt{floor}).

3 Strings and Characters

Strings are written as you might expect, with double quotes, and have the SML type \texttt{string}.

\["Hello, world!\n" : \texttt{string}\]

Characters are written as a hash \# immediately followed by a single-character string.

\[
\begin{align*}
\texttt{"a"} & : \texttt{char} \\
\texttt{"\n"} & : \texttt{char}
\end{align*}
\]

Unlike C, characters are \textit{not} a sort of integer, so you can’t do arithmetic with characters.

The function \texttt{explode} converts a string to a list of characters, and the function \texttt{implode} does the opposite.

4 Booleans

The type \texttt{bool} contains exactly two values:

\[
\begin{align*}
\texttt{true} & : \texttt{bool} \\
\texttt{false} & : \texttt{bool}
\end{align*}
\]

The comparison operations \texttt{<}, \texttt{<=}, \texttt{>=}, and \texttt{>} take two integers, two reals, two strings, or two characters, and return a boolean result.

\[
\begin{align*}
3 > 5 & : \texttt{bool} \\
\texttt{"a"} <= \texttt{"z"} & : \texttt{bool}
\end{align*}
\]

\(^1\)The language standard does not specify how large such integers can be; the SML/NJ implementation represents integers as 31-bit numbers.
Inequality is written $\langle\rangle$ (not $\neq$), and equality is written with a single equals sign, $=$.

\[
3 \langle\rangle 5 : \text{bool} \\
"a" = "a" : \text{bool}
\]

The operators for negation, logical-or, and logical-and are not, andalso, and orelse.

\[
\text{not} (3 > 5) : \text{bool} \\
(4>2) \text{ andalso } (2=3) : \text{bool} \\
(3<2) \text{ orelse } (5<>3) : \text{bool}
\]

The operators andalso and orelse are short-circuiting; if the first argument to andalso is false, then the second argument won’t be evaluated, and similarly if the first argument to orelse is true then the second won’t be evaluated.

Boolean values can be used by if-then-else or conditional expressions:

\[
\text{if } ((3>4) \text{ orelse } (5<2)) \text{ then } "yes" \text{ else } "no" : \text{string} \\
(\text{if } (4 > 5) \text{ then } 3 \text{ else } 5) + 1 : \text{int}
\]

Because these conditionals are expressions returning values, they correspond to ... ? ... : ... in C++ or Java rather than to if-then. Such an expression must always return a value no matter whether the test turns out true or false, so the else branch cannot be omitted.

The typechecker requires that the tests in conditional expressions have type bool and that the then and else branches have the same type.

## 5 Pairs, Tuples, and Records

### 5.1 Pairs and Tuples

An ordered pair whose first component has type $t_1$ and whose second component has type $t_2$ has the SML type $t_1 \times t_2$. For example:

\[
(3, "three") : \text{int} \times \text{string} \\
(4.0, 5.0) : \text{real} \times \text{real}
\]

This can be extended to ordered triples, ordered quadruples, etc.

\[
(1, #"3", #"3") : \text{int} \times \text{char} \times \text{string}
\]

The following values all have different types, however:

\[
(1, 2, 3) : \text{int} \times \text{int} \times \text{int} \\
((1, 2), 3) : (\text{int} \times \text{int}) \times \text{int} \\
(1, (2, 3)) : \text{int} \times (\text{int} \times \text{int})
\]

The first is a triple, while the last two are pairs (of different types). Parenthesization of types matters.

\[2\text{The operator } = \text{ does not work for equality between two real numbers. There is a function for comparing floating-point values, Real.==, but because of floating-point roundoff and other imprecision, it’s almost always a bad idea to directly compare two floating-point numbers for equality anyway.}[/2}
5.2 Records

If tuples have many components and/or many of these components have the same type, it is easy to write code with elements in the wrong order. In such cases, a record may be more appropriate. A record is very similar to a tuple, except that each component has a label (a name). Record expressions are written with curly braces around a sequence of named components. The types of records are similar, except that they give types for each label in the record. For example, a 2-dimensional point could be represented as a record with two real components labeled x and y:

\[
\{ x = 3.14, y = -2.0 \} : \{ x : \text{real}, y : \text{real} \}
\]

When writing a record or its type, the components can be listed in any order; it doesn’t matter. Hence,

\[
\{ \text{name} = "Pat", \text{age} = 20, \text{occupation}"=\text{"student"} \} : \\
\{ \text{occupation} : \text{string}, \text{name} : \text{string}, \text{age} : \text{int} \}
\]

The components of a tuple or record can be accessed via pattern-matching (see below).

6 Lists

In their most familiar form lists are written as in rex with square brackets and commas between elements. Unlike in rex, lists in Standard ML must be homogeneous: all elements of a list must have the same type. When every element of the list has type \( t \), the type of the entire list is written \( t \text{ list} \). For example:

\[
[1,2,3,4] : \text{int list} \\
[(0,"black"), (1,"brown"), (2,"red"), (3,"orange"), (4,"yellow"), (5,"green"), (6,"blue"), (7,"violet"), (8,"grey"), (9,"white")]: (\text{int} \ast \text{string}) \text{ list}
\]

Recall the inductive definition of lists from CS 60: every list is either empty, or else it consists of a single element (the “head” or “first”) and another list (the “tail” or “rest”). This is how SML lists are really defined. The empty list is written nil, and given an element \( h \) and a list \( t \), we can construct a new list \( h :: t \) whose first element is \( h \) and whose remaining elements are contained in the list \( t \). (Of course since lists are homogenous the types of the head element and the elements of the tail must all be the same.) The infix operator :: is pronounced “cons”.

The expression \( h :: t \) has exactly the same meaning as the rex expression \( [h | t] \). Be very careful not to get these notations confused. The code \( [h :: t] \) means something different from \( h :: t \) in SML. (The former is a list of length one whose single components is a list; neither of these properties is necessarily true of the latter.)
Thus, the list syntax with square brackets and commas is simply a handy abbreviation for uses of \texttt{nil} and \texttt{::}. For example,

\[
\begin{align*}
[] & = \texttt{nil} \\
[1,2] & = (1 :: [2]) = (1 :: (2 :: \texttt{nil}))
\end{align*}
\]

You can use either syntax, depending on what is convenient.

SML provides some useful built-in functions for lists, including \texttt{rev} for reversing lists and the infix operator \texttt{@} for appending two lists.

\[
\begin{align*}
\texttt{rev} [1,2,3] & = [3,2,1] \\
[1,2,3] \ @ [4,5,6] & = [1,2,3,4,5,6]
\end{align*}
\]

Note that \texttt{rev} and \texttt{@} do not modify their arguments, but instead create new lists.

Be very careful not to confuse \texttt{::} with \texttt{@} as the former prepends a \textit{single element} onto the front of a list of such elements, while the latter combines two lists of the same type.

## 7 Pattern Matching

Pattern-matching in SML is very similar to what you saw in rex and Prolog. Given a value and a compatible pattern, the value may or may not \textit{match} the pattern. SML allows many sorts of patterns:

- A variable is a pattern that matches any value. When an entire pattern matches a value, then each variable occurring in the pattern is bound to (i.e., defined to be) the corresponding part of that value. Note that matching a value against a variable, say \texttt{x}, is neither an assignment to an existing variable \texttt{x} nor a test to see whether the value is equal to the contents of an existing variable \texttt{x}, but rather initializes a \textit{new} local variable named \texttt{x}. Pattern-matching against a variable always succeeds.

  Unlike in rex and Prolog, patterns in SML must be “linear”: no pattern can contain the same variable multiple times.

- The \textit{wildcard} or underscore pattern \_ matches any value, but does not define a variable; it means “don’t-care”. Multiple wildcards may appear within a single pattern, and each can match a different value.

- A constant integer, character, string, or boolean can be used as a pattern; it matches exactly that integer, character, etc.

- A tuple pattern is written simply as a tuple of patterns. It matches a tuple if the components of the pattern match the corresponding components of the tuple. For example, the pattern \((3,\_)\) matches any pair of values whose first component is the integer 3. The pattern \((\texttt{x},y,z)\) matches any triple and gives the name \texttt{x} to the first component, \texttt{y} to the second component, and \texttt{z} to the third component. Record patterns can be written similarly as a record of patterns, e.g., \{\texttt{name=x,age=20,occupation=z}\}, which matches records having components with those three names where the \texttt{age} field contains the integer 20.
• List values can be matched with lists of patterns; either with the square bracket syntax as in \([x,y,z]\) (which matches any list of length exactly three and binds these three elements to variables) or the nil and :: syntax as in \((x::(y::\_))\), which matches any list with length \(\geq 2\) and gives the names \(x\) and \(y\) to the first two elements. It is usually a good idea to put parentheses around patterns involving ::.

• Datatype constructors may appear in patterns. (See the discussion of datatypes below.)

• Any pattern can be annotated with a type, to help guide type inference (see below) in figuring out what sort of value is being matched against. Thus, the pattern \(x:int\) can be used to match integers, while the pattern \((x:int,y:bool)\) can be used to match any \(int*bool\) pair, and bind the names \(x\) and \(y\) to the two components.

Pattern-matching comes up in several contexts including definitions and functions (see below), but can directly used in a case expression. This is analogous to switch in C, but the cases are specified using patterns rather than integer constants. The first case whose pattern matches the value is always chosen. For example, assuming that \(lst\) is a variable of type \(int list\):

\[
(case \text{lst} \text{of}
\quad [] \Rightarrow \text{"empty"}
\quad [\_] \Rightarrow \text{"length one"}
\quad [3,\_] \Rightarrow \text{"length two, starting with three"}
\quad _ \Rightarrow \text{"unknown"}) : \text{string}
\]


It is usually a good idea to put parentheses around case statements as in this examples; in rare cases the compiler can get confused otherwise. Also, note that the vertical bar is used to separate the cases, so there’s no vertical bar before the first.

Because SML is strongly typed, the type of the value being matched determines which patterns are allowed. For example, if the value being tested in a case instruction has type \(int\), then only variable, wildcard, or constant-integer patterns can appear in the case. Conversely, the typechecker will reject any a case statement that tries to do one thing if given a boolean and another thing if given an integer, or which does one thing given a pair and another thing given a triple.

The compiler also checks for redundant matches (those which will never be picked) and inexhaustive matches (where some values might not match any of the cases). At run time, if there is no pattern which matches the value being tested then the case expression raises a Match exception.

8 Functions

Standard ML functions are first-class values like integers and strings; they can be passed as arguments to functions, can be returned from functions, can be the value of a variable, and so on. Every function in SML takes exactly one argument and returns exactly one result. The type of a function which expects an argument of type \(t_1\) and returns a result of type \(t_2\) is written \(t_1 \rightarrow t_2\).
A function value can be written \( \text{fn } \text{pattern } \Rightarrow \text{expression} \). When applied, the given argument is matched against the pattern, and then the expression making up the body of the function is evaluated. For example,

\[
\text{fn } x: \text{int } \Rightarrow x + 1
\]

is a value of type \( \text{int } \Rightarrow \text{int} \), namely the successor function on integers. Similarly,

\[
\text{fn } (x: \text{int}, y: \text{bool}) \Rightarrow (y, x)
\]

has type \( \text{int*bool } \Rightarrow \text{bool*int} \) (in types, \(*\) binds more tightly than \(\Rightarrow\)) and is a “swapping” function that takes a pair and returns a new pair with the components in the opposite order. This is still a one-argument function; the one argument is a pair, however, and by using pattern-matching notation we can assign names to the two components of that single pair.

Functions are applied by “juxtaposition”; two expressions written next to each other are interpreted as function application. For example,

\[
(\text{fn } x: \text{int } \Rightarrow x + 1) \ 3
\]

is an application of the successor function to the argument 3; unsurprisingly, the value of this application expression is 4. Since one can always put a pair of grouping parentheses around any expression, this application could also be written

\[
(\text{fn } x: \text{int } \Rightarrow x + 1) \ (3)
\]

A more common case of application is when we have a variable, say \( f \), whose value is a function. If

\[
f : \text{int } \Rightarrow \text{string}
\]

then we can say \( f \ 3 \) or \( f(3) \) or \( (f)(3) \) or \( f \ (2+1) \) to apply this function to the argument 3 and get back a \text{string}. Application has higher precedence than the other operators, though, so the similar-appearing expression \( f \ 2+1 \) is understood to mean the ill-typed expression \( (f \ 2) + 1 \).

9 Type Inference

Functional languages work not by assigning new values to existing variables, but by allocating new variables to hold new values. With all these variables being defined, it would be relatively painful to have to give a types of every single variable appearing in definitions or patterns. Fortunately, SML requires that implementations do \text{type inference}, which is the process of figuring out what the types for all of the variables need to be. Thus, if you write

\[
\text{fn } x: \text{int } \Rightarrow x + 1 \ 3
\]

as

\[
\text{fn } x: \text{int } \Rightarrow (x + (1 \ 3))
\]

and since you can’t apply the integer 1 to another integer 3, a type error would be reported.
the computer sees that you are adding the function argument x to an integer, concludes that the variable x must range over integers, and concludes that this expression has type \texttt{int -> int}.

10 Definitions

10.1 \texttt{val}

The general form of variable definition in SML is

\[
\texttt{val \ pattern = expression}
\]

which evaluates the expression and then matches it against the pattern to define any new variables. (If the pattern fails to match, a \texttt{Bind} exception is raised at run-time; the compiler will warn you about a non-exhaustive match if it thinks this might happen.) For example,

\begin{verbatim}
val twice = (fn (n:int) => n+n)
val swapint = fn (x:int,y:int) => (y,x)
val two = twice 1
val (four, three) = swapint (two+one, twice 2)
\end{verbatim}

Remember, these definitions are initializations of \textit{new} variables, rather than assignments!

Because the right-hand-side is evaluated before the pattern-matching and variable definition occurs, the definition

\begin{verbatim}
val fact : int->int = fn n => if (n = 0) then 1 else n * fact(n-1)
\end{verbatim}

doesn’t work. The function value on the right-hand-side won’t typecheck because \texttt{fact} isn’t defined yet. To specify that you want a \textit{recursive} definition that can refer to the variable being defined you need to say \texttt{val rec} instead of \texttt{val}:

\begin{verbatim}
val rec fact : int->int = fn n => if (n = 0) then 1 else n * fact(n-1)
\end{verbatim}
10.2 fun

10.2.1 Overview

The above definition of factorial is pretty ugly. Fortunately, SML provides an alternative syntax for val rec. The first two definitions above can be written

    fun twice (n:int) = n+n
    fun swapint (x:int,y:int) = (y,x)

and the factorial function can be written

    fun fact(n:int) : int = 
      if (n = 0) then
        1
      else
        n * fact(n-1)

Note the keyword is fun (with a “u”) instead of val, and the argument pattern appears on the left-hand-side of the equals sign. Note also that if you want to specify types (optional, due to type inference) the type of the argument and the type of the function’s result are specified separately.

Remember, definitions using fun are just convenient syntax for defining a variable to have a function as its value! The compiler internally converts every such function into equivalent code written using val rec.

10.2.2 Clausal Definitions

Even better, definitions of functions can specify multiple rules as in rex, by giving a sequence of cases (called clauses) separated by vertical bars. When the function is applied, the first clause whose pattern matches the argument is chosen. For example, yet another way to write factorial would be

    fun fact 0 = 1
    | fact n = n * fact(n-1)

Note that the name of the function must be repeated for each case.

As another example,

    fun sumlist [] = 0
    | sumlist (n::ns) = n + sumlist ns

which defines the variable

    sumlist : int list -> int

as a function which sums lists of integers.
10.2.3 Curried Functions

There are two obvious ways in SML to write a function of two arguments, corresponding to two different “calling conventions”. Either we can supply both the arguments together at the same time, or we can supply the arguments separately, first one and then (at some later point) the other. The former calling convention can be implemented as a function that takes a pair, e.g.,

```sml
fun add_u (x,y) = x+y
```
and the latter can be implemented by taking just one of the arguments, and returning a function that takes the second argument and does the actual computation:

```sml
fun add_c x = (fn y => x+y)
```

Here

```sml
add_u : int * int -> int
add_c : int -> (int -> int)
```

and the following two expressions both result in the value 4:

```sml
add_u (1,3) : int
(add_c 1)(3) : int
```

The advantage of the latter form (called the curried form) is that we can apply the two arguments at completely different points in the program. For example, we could define

```sml
val succ : int->int = add_c 1
```
and then evaluate `succ 3` to get 4. Since curried functions are very common, SML provides special syntax; `fun` definitions may have multiple patterns separated by whitespace, representing arguments to supplied sequentially. (A very similar syntax was also available in rex.) An equivalent definition of `add_c` is thus

```sml
fun add_c x y = x+y
```

Curried arguments and multiple clauses can be combined, e.g.,

```sml
fun add_c' 0 y = y
| add_c' x y = x+y
```

which defines `add_c' : int->int->int` to avoid the addition if the first argument is zero. A more interesting example is the built-in function

```sml
map : ('a -> 'b) -> (('a list) -> ('b list))
```

which takes (in successive applications) a function and a list, and returns the results from applying the function to every element of the list. This could be defined as:

```sml
fun map f [] = []
| map f (x::xs) = (f x) :: (map f xs)
```

Arrows in types are right-associative, so the type of `map` could also be written with one fewer pair of parentheses as `('a -> 'b) -> ('a list) -> ('b list)`. Deciding whether a function definition should be curried or not depends on how it is to be used, and is to a large extent a matter of taste.
10.2.4 Mutual Recursion

If one wants to write mutually-recursive functions using `val rec` or `fun` this can be done by writing the definitions in sequence, using `and` instead of `val rec` or `fun` after the first. For example,

```plaintext
fun even(n) = if (n=0) then true else odd(n-1)
and odd (n) = if (n=1) then true else even(n-1)
```

or equivalently

```plaintext
fun even(n) = (n=0) orelse odd(n-1)
and odd (n) = (n=1) orelse even(n-1)
```

11 Local Definitions

It is often useful to define variables while inside a larger expression, such as the body of a function. The expression `let definitions in expression end` allows the creation of local variables; to evaluate a `let`-expression, first the definitions are evaluated, then the given expression is evaluated using those definitions, and finally these local definitions are discarded and the value of the body is returned. So, the expression

```plaintext
2 + let
  val fun thrice(n) = 3*n
  val y = 4
  in
    thrice(y)
  end
end
```

evaluates to 14.

12 Polymorphism

For some definitions, it’s not obvious what the type of the defined variable should be. For example, consider

```plaintext
fun length [] = 0
| length (x::rest) = 1 + length rest
```

This function could reasonably be applied to lists of integers, lists of strings, lists of pairs of functions, or any other sort of list. In fact, for any type `t`, this function could have type `t list -> int`.

SML handles this by allowing type variables in types. Such variables always have a name starting with a single quote, such as `'a` or `'b`. These are called type variables because they range over types like `int` or `((char*bool) list) list`. Given the above definition, SML will conclude that
length : 'a list -> int

i.e., for every type 'a, the function length can be applied to a value of type 'a list to obtain an integer. Thus, we have

length [1,2,3] : int
length [true, false] : int
length [fn x=>x+1, fn x=>x, fn x=>x-1] : int

and so on.

Similarly, the built-in function rev has type 'a list -> 'a list (meaning that it can be applied to a list of any type, and returns a list of the same type) and the definition

fun swap(x,y) = (y,x)

yields

swap : 'a*'b -> 'b*'a

meaning that for any types 'a and 'b (possibly but not necessarily different) the swap function can be applied to a value of type 'a*'b to obtain a value of type 'b*'a. For example,

swap (3,true) : bool * int
swap (5.0, 2.0) : real * real

As one more example, the empty list nil has type 'a list.

Warning: There is a restriction in SML that definitions must be values (i.e., a function or a constant like nil) to be given polymorphic types. If you get a type inference error/warning about the value restriction, this is what went wrong; you have an expression like rev nil which could be given infinitely many different types, but isn’t allowed to be polymorphic (since function applications aren’t values). This can usually be fixed by explicitly specifying the type you want (e.g., val lst : int list = rev nil).

13 Type Abbreviations

Sometimes types in SML can get large; it can be convenient to give a short name to a big type, the way that typedef is used in C. This is easy to do; for example, the definition

type striple = string*string*string

says that the name striple can be used interchangeably with its definition, so that we can later say

val x : striple = ("a","b","c")

instead of

val x : string*string*string = ("a","b","c")
Type definitions can also be parameterized. For example, the definition

\[
type 'a \text{ pair} = 'a \times 'a
\]

allows us to use \texttt{int pair} interchangeably with \texttt{int*int}, and similarly to write \texttt{striple pair} instead of

\[
\text{(string*string*string) *(string*string*string)}.
\]

Type abbreviations never define new types; they can be thought of as a kind of macro. They may \textit{not} be recursively defined.

14 Datatypes

14.1 \texttt{datatype}

SML datatypes correspond to what some other languages call “tagged unions” or “disjoint unions”. The elements of a datatype are values with tags attached. For example, the definition

\[
\text{datatype intOrReal = Int of int}
\text{ | Real of real}
\]

actually defines three names:

- It defines a \textit{new} type \texttt{intOrReal}.
- It specifies that there are two sorts of values of type \texttt{intOrReal}: such values either contain the tag \texttt{Int} and an integer value, or contain the tag \texttt{Real} and a floating-point value. These tags are called \textit{datatype constructors}, or constructors for short. We can create values of this type by tagging integers or reals with the appropriate constructor:

\[
\begin{align*}
\text{Int 3} : & \text{ intOrReal} \\
\text{Int (~5)} : & \text{ intOrReal} \\
\text{Real 4.2} : & \text{ intOrReal} \\
[\text{Int 3, Int ~5, Real 4.2}] & \text{ : intOrReal list}
\end{align*}
\]

Constructors can also be used in pattern-matching. Recalling that the function \texttt{real} converts integers to floating-point numbers, we can write definition

\[
\begin{align*}
\text{fun add(Int n, Int m) = Int(n+m)} \\
| \text{add(Int n, Real r) = Real(real(n) + r)} \\
| \text{add(Real r, Int n) = Real(r + real(n))} \\
| \text{add(Real r, Real s) = Real (r+s)}
\end{align*}
\]

to define a function \texttt{add : intOrReal*intOrReal -> intOrReal}.
A datatype can have arbitrarily many constructors, some of which may not even need to be attached to a value. For example,

```plaintext
datatype color = Red
  | Green
  | Blue
  | RGB of {redvalue: int, bluevalue:int, greenvalue:int}
```

defines a type color whose values are either the constructor Red by itself, the constructor Green by itself, the constructor Blue by itself, or a three-component record tagged with the constructor RGB. Thus, for example

```
[Red, RGB{redvalue=255, bluevalue=255, greenvalue=0}, Blue]
```

: color list

If none of the constructors have associated data, then we get something analogous to an “enumerated type” in other languages, e.g.,

```plaintext
datatype day = Monday | Tuesday | Wednesday | Thursday
  | Friday | Saturday | Sunday
```

Note, however, that the values Monday through Sunday of this new type day cannot be used as integers.

### 14.2 Recursive Datatypes

Datatypes can be recursively defined. For example, we can define a type mimicking integer lists as follows:

```plaintext
datatype myIntList = Empty
  | NonEmpty of (int * myIntList)
```

which says that a value of type myIntList is either the constant Empty or is tagged NonEmpty and has a pair containing an integer (the head) and another myIntList (the tail). Thus, given the definition

```plaintext
fun myIntListLength Empty = 0
  | myIntListLength (NonEmpty(_,rest)) = 1 + myIntListLength(rest)
```

that defines myIntListLength : myIntList -> int we have

```
myIntListLength (NonEmpty(3, NonEmpty(4, Empty))) : int
```

with this expression evaluating to 2.

Datatypes can be used to define many other inductively-defined data structures as well (e.g., trees, logical propositions, C++ syntax).

Mutually-recursive datatype definitions can be connected with and, just as mutually-recursive functions are.
14.3 Parameterized Datatypes

Finally, datatypes can be parameterized, just as type definitions can. A particularly useful definition that is already predefined in SML is

\[
\text{datatype } 'a \text{ option } = \text{NONE} \\
| \text{SOME of } 'a
\]

Thus we have

\[
\begin{align*}
\text{NONE} &: 'a \text{ option} \\
\text{SOME 3} &: \text{int option} \\
\text{SOME true} &: \text{bool option} \\
\text{SOME (3,4)} &: (\text{int*int}) \text{ option} \\
(3, \text{SOME 4}) &: \text{int * int option}
\end{align*}
\]

The option type is useful when we may or may not have a value. For example, the function to convert a string to an integer is

\[
\text{Int.fromString : string } \rightarrow \text{ int option}
\]

This function returns \text{SOME } n \text{ if the given string contains a recognizable integer } n \text{ and returns } \text{NONE} \text{ otherwise.}

The built-in type list could have been defined as

\[
\text{datatype } 'a \text{ list } = \text{nil} \\
| :: \text{ of } 'a \ast 'a \text{ list}
\]

except that the cons operator :: is written infix, i.e., as \(h::t\) instead of ::(\(h,t\)).

15 Exceptions

Exceptions in SML behave generally the same as those you have seen in C++ and Java. The definition of new exceptions is reminiscent of the syntax for datatypes, e.g.,

\[
\text{exception Oops}
\]

or

\[
\text{exception BadValue of int}
\]

An exception is thrown by using \text{raise} expressions:

\[
\text{raise Oops}
\]

or

\[
\text{raise BadValue("1")}
\]

Finally exceptions are caught using \text{handle} expressions. The code
which returns the value of $f(17)$ if this yields a value, returns the answer 0 if $f(17)$ raises the Oops exception or the exception BadValue($\sim 3$), returns the answer $n$ if $f(17)$ raises the exception BadValue($n$) for $n \neq 3$, and returns the answer 42 if any other exception was raised. Of course not every exception handler has to catch every possible exception.

16 Modules

16.1 Structures

A structure is a collection of definitions (of values, types, other structures, etc.), bracketed by struct...end. It is somewhat analogous to a package in Java, except that once a structure is defined one cannot go back and add new components. Here’s one, which defines an implementation of sets of integers using lists.

```
struct
type set = int list
val empty = []
fun insert(x,s) = x::s
fun member(x,[]) = false
  | member(x,h::t) = (x=h) orelse member(x,t)
end
```

16.2 Signatures

A signature is a specification for a structure; it contains a sequence of specifications (descriptions of defined names) bracketed by sig...end. Here’s a signature describing a generic implementation of sets of integers:

```
sig
type set
val empty : set
val insert : int * set -> set
val member : int * set -> bool
end
```

A signature is said to “match” a structure if the structure contains definitions satisfying all the specification in the signature. A structure can contain more items than the signature and still match.
16.3 Module Definitions

Signatures and structures are not first-class values, so they can’t be bound to variables. We can still give them names, though; we just can’t use val:

```sml
structure Set = struct
  type set = int list
  ...as above...
end
signature SET = sig
  type set
  ...as above...
end
```

Once we have a name for a structure, we can refer to its components by name, e.g., the type Set.set or the value Set.empty.

16.4 Information Hiding

We can use a signature to hide everything about a structure except what is explicitly stated in a signature it matches by using the :> operator. Thus, after the definitions

```sml
signature SET = sig
  type set
  val empty : set
  val insert : int * set -> set
  val member : int * set -> bool
end
structure Set :> SET = struct
  type set = int list
  val empty = []
  val insert(x,s) = x::s
  fun member(x,[]) = false
    | member(x,h::t) = (x=h) orelse member(x,t)
end
```

the type Set.set is treated as abstract; we don’t get to use the fact that sets are implemented as lists because we can’t tell this from the SET signature. Thus we know that Set.empty has type Set.set, and can still write Set.add(3, Set.empty) but the expression 3 :: Set.empty will be rejected by the typechecker. This ensures that we could change the implementation of sets (to use ordered binary trees, for example) and know that all code using the Set module would still work.

17 The Rest of SML

Features not described in this document include references and assignment, input-output, and functors (parameterized structures).
A  Appendix: Some Built-In Functions

A.1  Type Conversions

real : int -> real
round : real -> int
ceil : real -> int
trunc : real -> int
chr : int -> char
ord : char -> int
explore : string -> char list
implode : char list -> string
concat : string list -> string
Int.toString : int -> string
Real.toString : real -> string
Bool.toString : bool -> string
Char.toString : char -> string

A.2  Higher-Order Functions

map : ('a -> 'b) -> ('a list) -> ('b list)  (curried)
o : ('a -> 'b) * ('c -> 'a) -> ('c -> 'b)  function composition (infix)

A.3  Miscellaneous

size : string -> int
^ : string*string -> string  string append (infix)
length : 'a list -> int
@ : 'a list * 'a list -> 'a list  list append (infix)
rev : 'a list -> 'a list
hd : 'a list -> 'a  head of a non-empty list
tl : 'a list -> 'a list  tail of a non-empty list
print : string -> unit

There are other useful functions in the built-in structures Int, Real, Char, String, List, Bool, and Math. (See the course web page for the signatures of these structures and the rest of the Standard Basis library.)