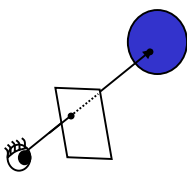


ray tracing

ray tracing

- simple ray casting
- recursive ray tracing
- modeling transforms
- cheap tricks
- optimizations

ray casting



- cast ray through pixel into scene
- find intersection point (if any) that is closest to eye
- compute luminance at intersection

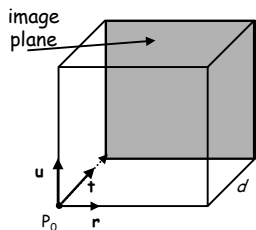
viewpoint orientation

3d world coordinates

- P_0 is the viewpoint
- u , t , and r are unit vectors (typically orthogonal) in the up, toward, and right directions

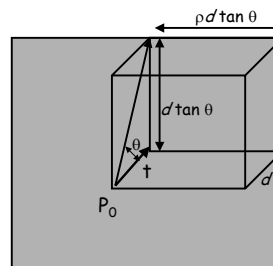


image plane



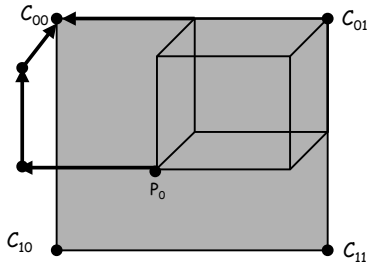
the image plane is orthogonal to t
a distance d from P_0

view window



- $P_0 + d\hat{t}$ is the center of the view window
- θ is the half-height angle of the frustum
- in world coordinates
 - image height is $H=2d\tan\theta$
 - image width is $W=p2d\tan\theta$ where p is the aspect ratio

corner coordinates



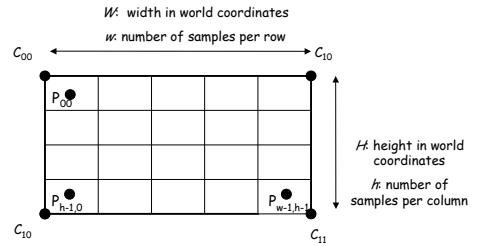
$$C_{00} = P_0 + -(W/2) \mathbf{r} + (H/2) \mathbf{u} + d \mathbf{t}$$

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pixel coordinates



$$P_{ij} = C_{00} + (i + \frac{1}{2})(W/w) \mathbf{r} - (j + \frac{1}{2})(H/h) \mathbf{u}$$

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ray tracing assignment

input specifications:

- $P_0 = (0,0,0)$
- $\mathbf{u} = \langle 0,1,0 \rangle$
- $\mathbf{t} = \langle 0,0,-1 \rangle$
- h
- w
- θ

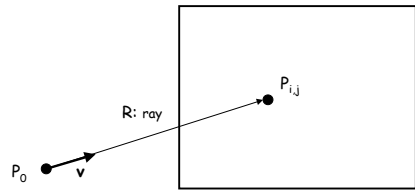
choose d so that
 $H/h = W/w = 1$

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casting rays



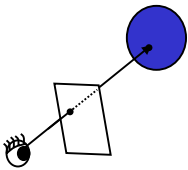
$$R = (P_0, \mathbf{v}) \text{ where } \mathbf{v} = (P_{ij} - P_0) / \|P_{ij} - P_0\|$$

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ray casting



- cast ray through pixel into scene
- **find intersection point (if any) that is closest to eye**
- compute luminance at intersection

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intersection

- sphere
- triangle
- box
- cylinder
- cone
- torus

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intersection

- sphere
- triangle
- box
- cylinder
- cone
- torus

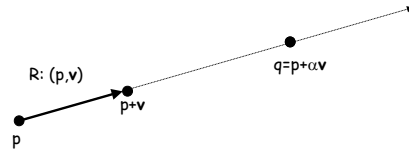
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ray: parametric form

a point q lies on $R=(p,v)$ iff $q=p+\alpha v$ for some $\alpha \geq 0$



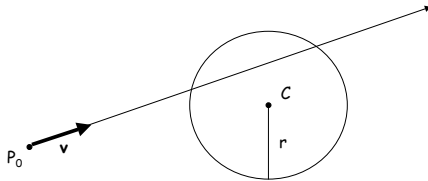
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sphere intersection

- Is there a point $q = P_0 + \alpha v$, $\alpha \geq 0$, such that $\|q - C\| = r$?
- If so, choose the one closest to P_0 .



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sphere intersection

- Is there a point $q = P_0 + \alpha v$, $\alpha \geq 0$, such that $\|q - C\| = r$?
- Assume there is and try to solve for α :

Let u be the vector from P_0 to C .

Let w be the vector from C to q .

Then $w = P_0 + \alpha v - C = \alpha v - u$ and

$$\begin{aligned} r^2 &= \|w\|^2 = w \cdot w = (\alpha v - u) \cdot (\alpha v - u) \\ &= \alpha^2 \|v\|^2 - 2\alpha(u \cdot v) + \|u\|^2 \end{aligned}$$

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sphere intersection

- Is there a point $q = P_0 + \alpha v$, $\alpha \geq 0$, such that $\|q - C\| = r$?
- Does the quadratic (in α)

$$\|v\|^2 \alpha^2 - 2(u \cdot v) \alpha + \|u\|^2 - r^2$$

have any real, non-negative roots?

•No: no intersection

•Yes: use smallest non-negative root for α

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intersection

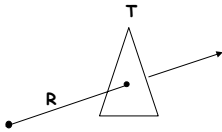
- sphere
- **triangle**
- box
- cylinder
- cone
- torus

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triangle intersection



- do R and T intersect?
- if so, where?

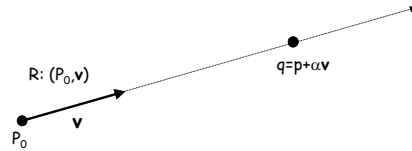
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ray: parametric form

a point q lies on $R=(p,v)$ iff $q=p+\alpha v$ for some $\alpha \geq 0$



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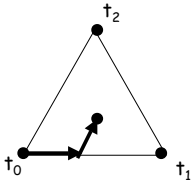
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triangle: parametric form

a point q lies on the triangle T iff $q=t_0+\beta u+\gamma w$ where

$u = t_1 - t_0$, $w = t_2 - t_0$, and $\beta \geq 0$, $\gamma \geq 0$, $\beta + \gamma \leq 1$

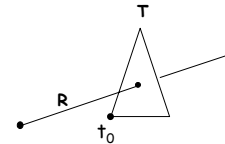


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triangle intersection



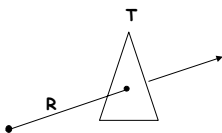
Find α, β, γ such that
 $P_0 + \alpha v = t_0 + \beta u + \gamma w$
 $\alpha, \beta, \gamma \geq 0$
 $\beta + \gamma \leq 1$

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triangle intersection - take 1



Find α, β, γ such that
 $P_0 + \alpha v = t_0 + \beta u + \gamma w$
 $\alpha, \beta, \gamma \geq 0$
 $\beta + \gamma \leq 1$

System of linear equations

$$P_{0x} + \alpha v_x = t_{0x} + \beta u_x + \gamma w_x$$

$$P_{0y} + \alpha v_y = t_{0y} + \beta u_y + \gamma w_y$$

$$P_{0z} + \alpha v_z = t_{0z} + \beta u_z + \gamma w_z$$

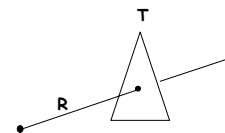
Invert the matrix. Yuck. Is there another way?

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triangle intersection - take 2



1. Find the intersection point (if any) of R and the plane containing T .
2. Find the parameterization of the intersection point in terms of β, γ .

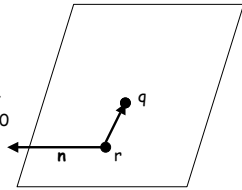
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plane

Let n be a normal to the plane.
 Let r be a point on the plane.
 Let q be an arbitrary point in space.
 Then q lies on the plane iff $n \cdot (q-r) = 0$



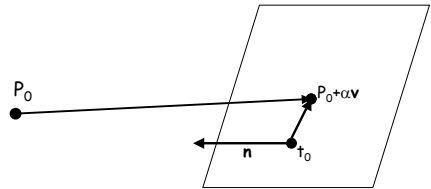
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plane intersection

Is there a point $P_0 + \alpha v$, $\alpha \geq 0$, such that $n \cdot (P_0 + \alpha v - t_0) = 0$?



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plane intersection

Is there a point $P_0 + \alpha v$, $\alpha \geq 0$, such that $n \cdot (P_0 + \alpha v - t_0) = 0$?

Assume there is and solve for α .

Let u be the vector from P_0 to t_0 .

Then $0 = n \cdot (P_0 + \alpha v - t_0) = n \cdot (\alpha v - u) = \alpha(n \cdot v) - n \cdot u$.

If $n \cdot v = 0$ then R is either parallel to plane or it lies in the plane.

If $n \cdot v \neq 0$ then $\alpha = (n \cdot u) / (n \cdot v)$.

If $\alpha \geq 0$ then R intersects the plane at point $P_0 + \alpha v$.

If $\alpha < 0$ then R does not intersect plane.

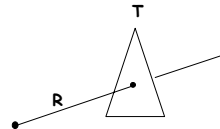
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triangle intersection - take 2

1. Find the intersection point (if any) of R and the plane containing T .
2. Find the parameterization of the intersection point in terms of β, γ .



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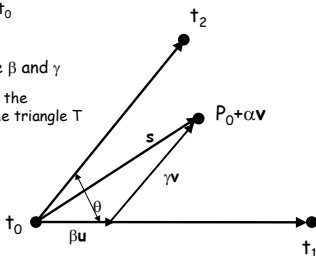
triangle intersection

recall $u = t_1 - t_0$ and $w = t_2 - t_0$

let $s = P_0 + \alpha v - t_0$

then $s = \beta u + \gamma w$ for some β and γ

if $\beta, \gamma \geq 0$ and $\beta + \gamma \leq 1$ then the intersection point lies in the triangle T



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triangle intersection

To find β and γ solve system of equations:

$$\beta(u \cdot w) + \gamma(w \cdot w) = s \cdot w$$

$$\beta(u \cdot u) + \gamma(w \cdot u) = s \cdot u$$

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intersection

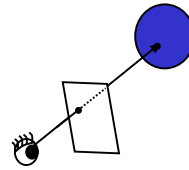
- triangle
 - sphere
 - box
 - cylinder
 - cone
 - torus
- } Etc.

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ray casting



- cast ray through pixel into scene
- find intersection point (if any) that is closest to eye
- **compute luminance at intersection**

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luminance

the luminance of a point on a surface depends on

- lights in scene
- material properties of surface
- geometry of scene

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luminance

for each channel we'll **approximate** the luminance at the intersection point as the sum of five terms

- emission
- ambient
- diffuse reflection
- specular reflection
- transmission (refraction)

We'll describe the terms for the red channel.

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luminance

for each channel we'll approximate the luminance at the intersection point as the sum of five terms

- **emission**
- ambient
- diffuse reflection
- specular reflection
- transmission

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red emission term

mer the material red emission

NOTE: terms shown in red are input parameters

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luminance

for each channel we'll approximate the luminance at the intersection point as the sum of five terms

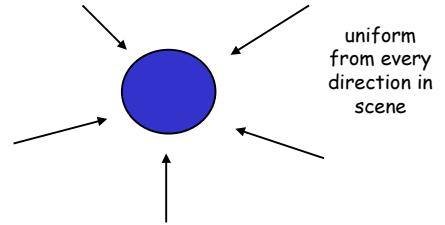
- emission
- **ambient**
- diffuse reflection
- specular reflection
- transmission

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ambient light



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red ambient term

the red ambient term is $ar * mar$ where

- ar is the red intensity of the ambient light
- mar is the response of the surface to red ambient light

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luminance

for each channel we'll approximate the luminance at the intersection point as the sum of five terms

- emission
- ambient
- **diffuse reflection**
- specular reflection
- transmission

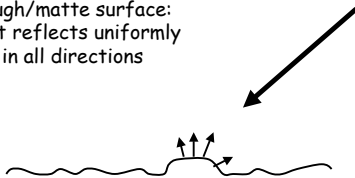
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diffuse reflections

rough/matte surface:
light reflects uniformly
in all directions



diffuse reflections provide the surface "color"

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red diffuse term

the red diffuse reflection term is $(1 - K_{trans}) \sum R_{L,D}$ where

- the summation is taken over all lights L
- $R_{L,D}$ is the intensity of the red, diffuse reflection of light L at the intersection point
- K_{trans} is the transmission coefficient of the surface

Note: terms in green are explained in subsequent slides

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$R_{L,D}$

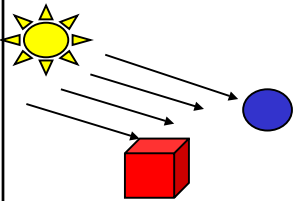
depends on

- type of light
- geometry of scene
- material properties of surface

types of lights

- directional light
- point light
- spot light

directional light



- light positioned at "infinity"; intensity and incident angle are constant for all points in scene
- specification
 - direction
 - red, green, and blue intensity

$R_{L,D}$ for directional light L

$R_{L,D} = 0$ if L is occluded

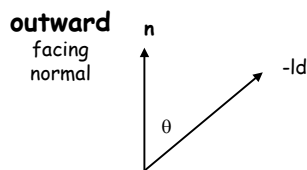
$$R_{L,D} = mdr \cdot lr \cdot \max(0, (\mathbf{n} \cdot -\mathbf{ld}))$$

otherwise

- mdr is the diffuse response of the surface material to red light
- lr is the red intensity of light L
- \mathbf{n} is the unit normal of the surface at the point of intersection
- \mathbf{ld} is a unit vector in direction the light falls

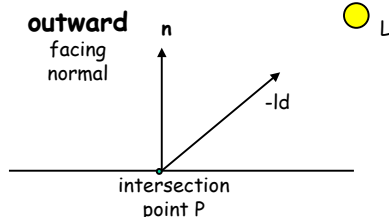
Note: you need to compute terms in blue

normal



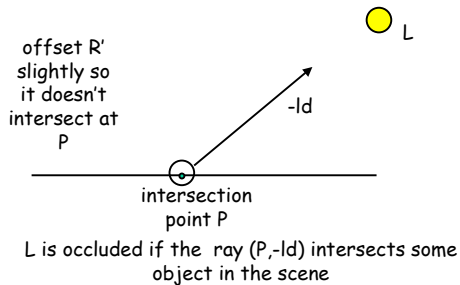
reflection decreases as θ increases. if $|\theta| > 90^\circ$ the light falls on the back of the surface and is not reflected

occlusion (shadows)



light L is occluded if the ray $\mathbf{R}=(\mathbf{P},-\mathbf{ld})$ intersects some object in the scene

occlusion: implementation

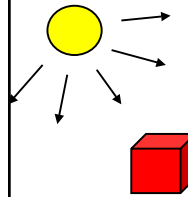


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point light



- light emanates uniformly in all directions
- specification
 - location in world coordinates
 - red, green, and blue intensity
 - how the light drops off with distance

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$R_{L,D}$ for point light L

$R_{L,D} = 0$ if L is occluded

$$R_{L,D} = A \cdot mdr \cdot lr \cdot \max(0, \mathbf{n} \cdot -\mathbf{ld})$$

otherwise

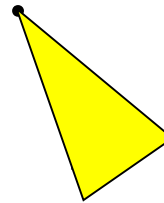
- mdr , lr , and \mathbf{n} are as previously defined
- \mathbf{ld} is the unit vector from the light position P_L to the intersection point
- $A = 1/(ca + la \cdot d + qa \cdot d^2)$ is the attenuation term, d is the distance between the light and the surface point

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spot light



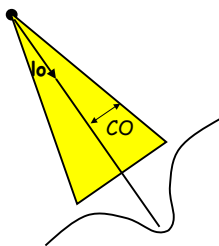
- light emanates in a cone
- specifications
 - location in world coordinates
 - red, green, and blue intensity
 - how the light drops off with distance
 - how light drops with angle from center

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spot light



- light emanates in a cone
- specifications
 - location in world coordinates
 - red, green, and blue intensity
 - how the light drops off with distance
 - how light drops with angle from center
 - light orientation vector \mathbf{lo}
 - cutoff angle CO
 - drop off exponent DQ_3

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$R_{L,D}$ for spot light L

$R_{L,D} = 0$ if L is occluded

$$R_{L,D} = A \cdot SP \cdot mdr \cdot lr \cdot \max(0, (\mathbf{n} \cdot -\mathbf{ld}))$$

where

- A , mdr , lr , and \mathbf{ld} are as previously defined
- SP is the "spot light effect"
 - $SP=0$ if angle between \mathbf{lo} and \mathbf{ld} is greater than CO
 - $SP = \max(0, \mathbf{ld} \cdot \mathbf{lo})^{DQ_3}$ otherwise

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luminance

for each channel we'll approximate the luminance at the intersection point as the sum of five terms

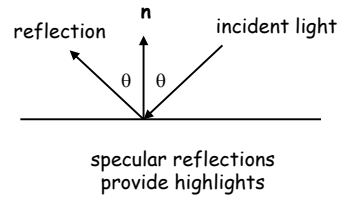
- emission
- ambient
- diffuse reflection
- **specular reflection**
- transmission

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specular reflections



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red specular

the red specular reflection term is

$\sum R_{L,S}$ where

- the summation is taken over all lights L
- $R_{L,S}$ is intensity of the red, specular reflection of light L at the intersection point (details to follow shortly)

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$R_{L,S}$

depends on

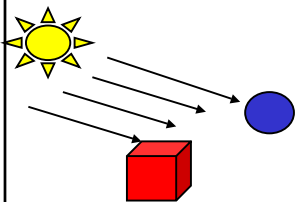
- type of light
- geometry of scene
- material properties of surface

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directional light



light positioned at infinity; intensity and incident angle are constant for all surface points in scene

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$R_{L,S}$ for directional light L

$R_{L,S} = 0$ if light is occluded

$R_{L,S} = msr \cdot lr \cdot \max(0, (-v \cdot dr))^{128} \cdot k_{spec}$ otherwise

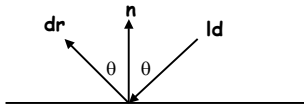
- msr is the specular response of the surface material to red light
- lr is as previously defined
- v is the direction of the incoming ray
- dr is a unit vector in the direction of reflection
- k_{spec} is the shininess constant

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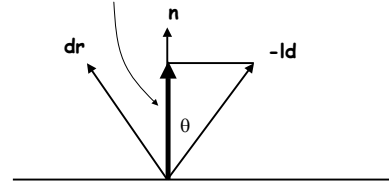
vector of reflection



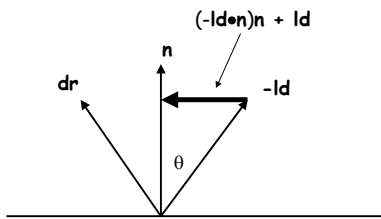
compute dr given n and Id
(all unit vectors)

vector of reflection

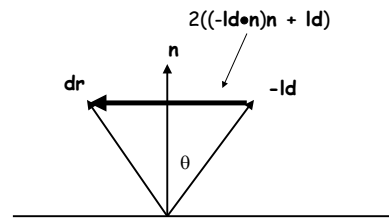
$(-Id \cdot n)n$ since n is a unit vector



vector of reflection

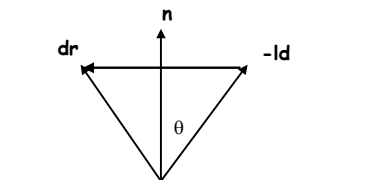


vector of reflection

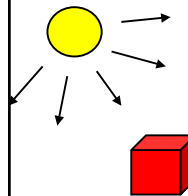


vector of reflection

$$dr = -Id + 2((-Id \cdot n)n + Id) = Id + 2(-Id \cdot n)n$$



point light



- light emanates uniformly in all directions
- specification
 - location in world coordinates
 - red, green, and blue intensity
 - how the light drops off with distance

$R_{L,S}$ for point light L

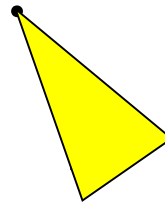
$R_{L,S} = 0$ if light is occluded

$$R_{L,S} = A \cdot msr \cdot lr \cdot \max(0, (-\mathbf{v} \cdot \mathbf{dr}))^{k_{spec}}$$

otherwise

- all terms are as previously defined

spot light



- light emanates in a cone
- specifications
 - location in world coordinates
 - red, green, and blue intensity
 - how the light drops off with distance
 - how light drops with angle from center

$R_{L,S}$ for spot light L

$R_{L,S} = 0$ if L is occluded

$$R_{L,S} = A \cdot SP \cdot msr \cdot lr \cdot \max(0, (-\mathbf{v} \cdot \mathbf{dr}))^{k_{spec}}$$

otherwise

- all terms are as previously described

luminance

for each channel we'll approximate the luminance at the intersection point as the sum of five terms

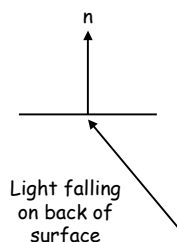
- emission
- ambient
- diffuse reflection
- specular reflection
- **transmission**

red transmission term

the red transmission term is $\sum R_{L,T}$
where

- the summation is taken over all lights L
- $R_{L,T}$ is the intensity of the red transmission of light L at the intersection point

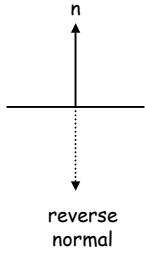
transmission



$$R_{L,T} = k_{trans} R_{L,D}^{back}$$

- where
- $R_{L,D}^{back}$ is the luminance of diffuse reflections on the back of the surface

transmission computation



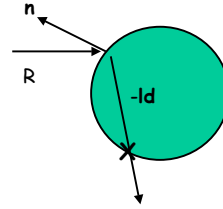
- compute $R_{L,D}$ with the reverse normal.
- scale by k_{trans}

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thick surfaces



all light falling on the back of the surface is occluded

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