

Harvey Mudd College  
Computer Science 80  
Logic for Computer Science  
Fall Semester 2002

Assignment #6 – Predicate Logic: Syntax, Semantics and Encoding  
Due 5:00pm, Tuesday December 10, 2002

1. Questions of Syntax:

- (a) Give the set of free variables and the set of bound variables of the formula:

$$\forall z(p(x) \wedge \forall x[\exists y\{q(y, f(z)) \Rightarrow r(g(g(w, x), f(y)))\}])$$

- (b) Give the result of substituting  $f(y)$  for the variable  $x$  in that formula.  
(c) Give the result of substituting  $f(y)$  for the variable  $w$  in the original formula (not in the answer from the last substitution).

2. Questions of Semantics:

For each of the following formulas (or, sets of formulas) show that they are both satisfiable and falsifiable, by giving interpretations for each case. Make sure you provide an assignment for quantified variables as well, where that appears necessary. Explanatory prose may also be helpful. For example:

- $\forall x(\exists y(l(y, x)))$

Satisfiable: Let the domain be the integers, and interpret  $l$  as the less-than relation. Clearly for every integer there is an integer less than it.

(Alternately, let the domain be any non-empty set, and interpret  $l$  as any equivalence relation on that set. Since an equivalence relation is reflexive, then each element is at least related to itself, if nothing else.)

Falsifiable: Let the domain be the non-negative integers and interpret  $l$  as the less than relation. Then if  $x$  is 0, there is no smaller natural number.

- (a)  $\exists x(\forall y(p(x, y)))$   
(b)  $\forall x(p(x) \Rightarrow q(x))$   
(c)  $(q(a) \wedge q(b) \wedge p(a)) \Rightarrow \forall x(p(x) \Rightarrow q(x))$   
(d)  $(jar(j) \wedge heated(j) \wedge bug(b) \wedge in(b, j)) \Rightarrow dead(b)$   
(e)  $[\forall x(plus(x, 0) \doteq x) \wedge \forall x(\forall y(plus(s(x), y) \doteq s(plus(x, y))))] \Rightarrow [plus(s(s(0)), s(s(0))) \doteq s(s(s(s(0))))]$

3. Questions of Encoding I:

Given the predicates defined in class for the beer-drinkers database example, provide existential formulas equivalent to the following queries:

- (a) What drinkers drink at least one beer served by the Hi-brow?
- (b) What drinkers drink all the beers served by the Hi-brow?
- (c) Who likes all the beers Hodas likes?
- (d) What bars does Hodas go to that serve all the beers he likes for less than \$3.00?
- (e) What bars are frequented by the drinkers who like all the beers that Hodas likes?

4. Questions of Encoding II:

Give first order encodings of each of the following facts. You should note that being a boletus, or fungus, or any particular sort of thing is a property, which is to say, a predicate. Also, you will need a constant to represent "this thing".

- (a) "Every fungus is either a mushroom or a toadstool."
- (b) "Every boletus is a fungus."
- (c) "Toadstools are poisonous, as are peach pits."
- (d) "A boletus is not a mushroom."
- (e) "This thing is a boletus."