Harvey Mudd College
Computer Science 80
Logic for Computer Science
Fall Semester 2002

Assignment #7 – Predicate Logic: Proof Theory

Due 5:00pm, Friday, December 13, 2002

1. Give Natural Deduction proofs of each of the following formulas (brackets are used in place of parentheses in some formulas to make the structure clearer):

   (a) $\exists x[\forall y(p(x,y))] \Rightarrow \forall y[\exists x(p(x,y))]$

   (b) $\exists x[p(x) \Rightarrow q(x)] \Rightarrow [\forall x(p(x)) \Rightarrow \exists x(q(x))]$

   (c) Prove that any boletus is poisonous. That is, from open assumptions:

   - $\forall x(f(x) \Rightarrow [m(x) \vee t(x)])$
   - $\forall x(b(x) \Rightarrow f(x))$
   - $\forall x([t(x) \vee pp(x)] \Rightarrow p(x))$
   - $\forall x(b(x) \Rightarrow \neg m(x))$

   prove the conclusion $\forall x(b(x) \Rightarrow p(x))$

   (d) (extra credit) $[\forall x(p(x)) \Rightarrow \exists x(q(x))] \Rightarrow \exists x[p(x) \Rightarrow q(x)]$

2. Give Gentzen Sequent Calculus proofs for each of the conclusions in the last problem, including the extra credit (which is not extra credit in this problem).