CS181a: Computer Animation

Mathematical Preliminaries

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Overview

- **Review of the graphics pipeline**
- Affine Spaces
- Barycenters

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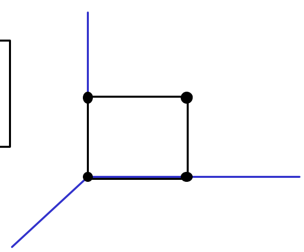
Graphics Pipeline

1. Build Primitives



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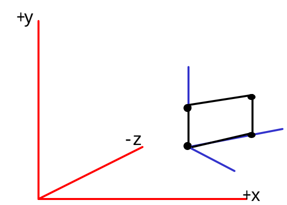
Graphics Pipeline

2. Assemble Scene



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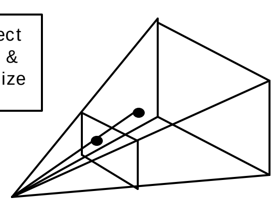
Graphics Pipeline

3. Project Scene & Normalize



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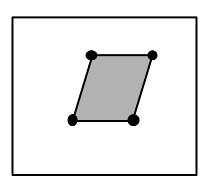
Graphics Pipeline

4. Scan Convert



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Graphics Pipeline

5. Display

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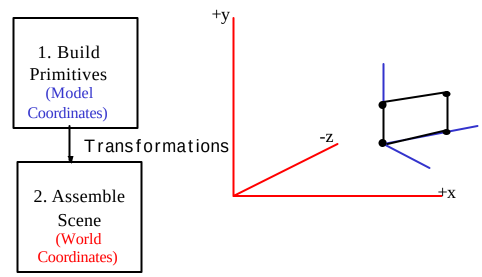
Graphics Pipeline 2



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Transformations

- Scale
- Rotate
- Translate

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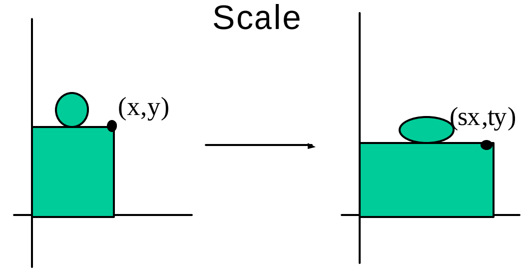
Scale



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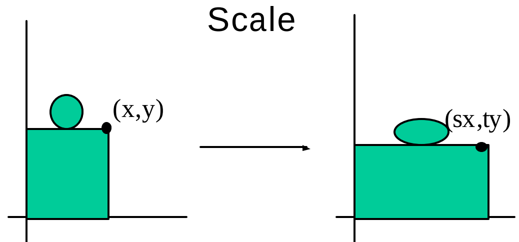
Scale



$$\begin{bmatrix} s & 0 \\ 0 & t \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} sx \\ ty \end{bmatrix}$$

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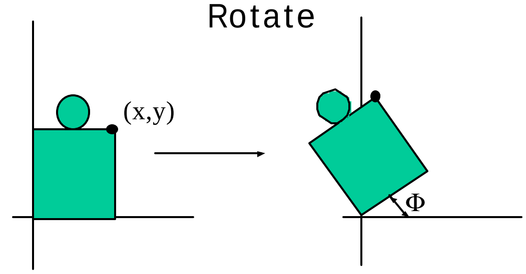
Rotate



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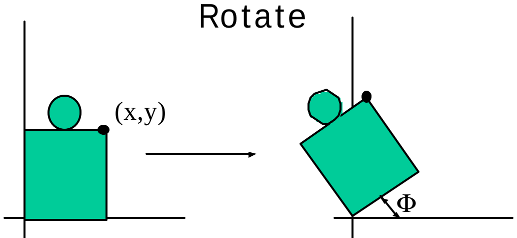
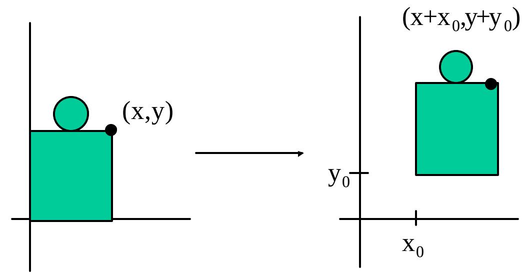
Rotate

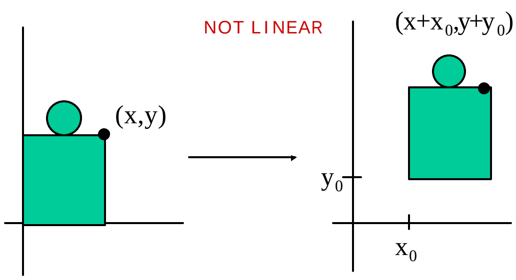


$$\begin{bmatrix}\cos\Phi & -\sin\Phi\\ \sin\Phi & \cos\Phi\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}x\cos\Phi - y\sin\Phi\\ x\sin\Phi + y\cos\Phi\end{bmatrix}$$

Translate



Translate



Solution

Homogenous Coordinates

Overview

- Review of the graphics pipeline
- **Affine Spaces**
- Curves Intro

Affine Spaces

- R^n: a set of points
- R^n: a Linear (Vector) Space
- R^n: an Affine Spaces
- Linear operators
- Frames
- Barycenters
- Affine operators

R^n: a set of points

R^n is the set of n-tuples of real numbers: i.e. $(x_1, x_2, \ldots, x_n)$ where x_i is in R

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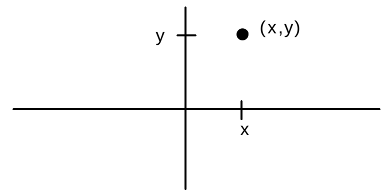
Points R^2 ={(x,y) where x,y are in R}



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R^n: a vector space

R^n is the vector space containing the vectors $\langle x_1, x_2, \ldots, x_n \rangle$ where x_i is in R.

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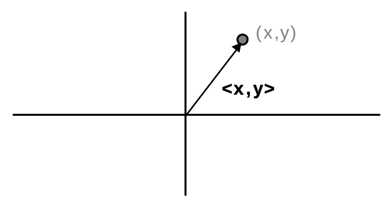
Vector space R^2: {<x,y> where x,y are in R}



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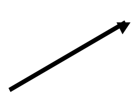
Vector

- Vector: has magnitude & direction but no position



- Good for representing change in position; i.e. go three miles northeast

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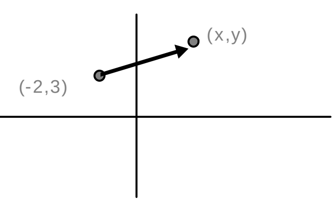
Vector space P^2: {vectors from (-2,3) to (x,y) where x,y are in R}



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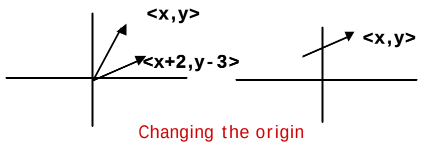
R^2 vs. P^2



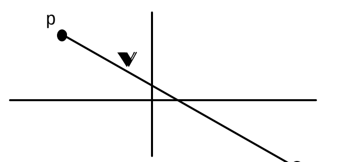
Changing the origin changes the naming convention

R^n: an Affine Space

R^n: points	+	R^n: vectors

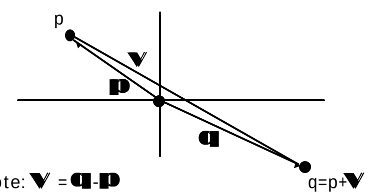
We need a consistent naming convention: $\langle x_1, x_2, \ldots, x_n \rangle$ is the vector defined from the point $(0,0,\ldots,0)$ to the point $(x_1, x_2, \ldots, x_n)$

Vectors Act On Points



p+**v** is the point you get to by moving **v** from the point p

In other words ...

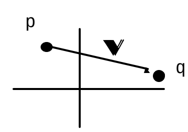


Note: **v** = **q**-**p**

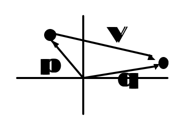
Summary

- p: the point
- **v**: the vector
- q=p+**v** is the point you get to by moving **v** from the point p
- Because of our naming convention **v** = **q**-**p**

Example



R^2: the points



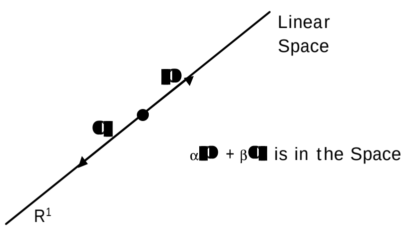
R^2: the vectors

Why bother?

Linear Spaces and Linear Operators are

NICE

Linear Space: closure under linear combination

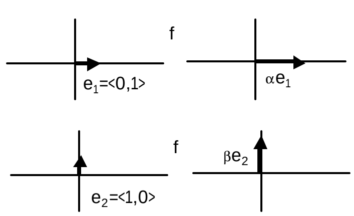


Functions (transformations) on Vectors

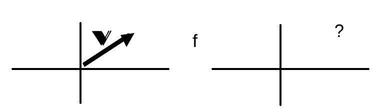
f is linear if

$$f(\alpha \mathbf{p} + \beta \mathbf{q}) = \alpha f(\mathbf{p}) + \beta f(\mathbf{q})$$

Example in R^2



What does f do to v



$\mathbf{v}$=<x,y> so f($\mathbf{v}$) = <αx, βy>

What does the linear operator f do to $\mathbf{v}$?

$\mathbf{v}=\langle x_1, x_2, \ldots, x_n\rangle = x_1 e_1 + x_2 e_2 + \ldots + x_n e_n$

$f(\mathbf{v}) = x_1 f(e_1) + x_2 f(e_2) + \ldots + x_n f(e_n)$

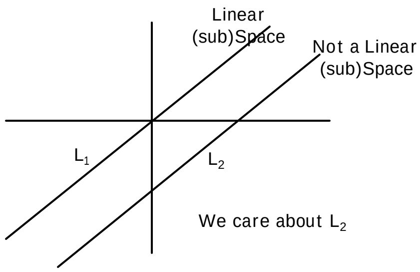
A linear operator is entirely characterized by its behavior on basis vectors

To reiterate:

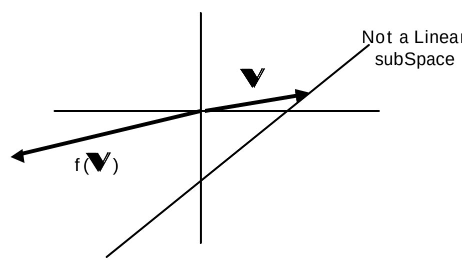
Linear Spaces and Linear Operators are

NICE

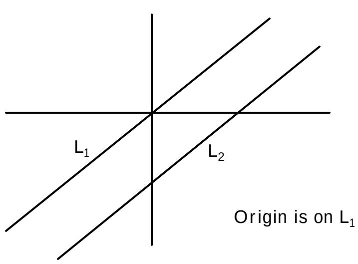
(sub)Spaces



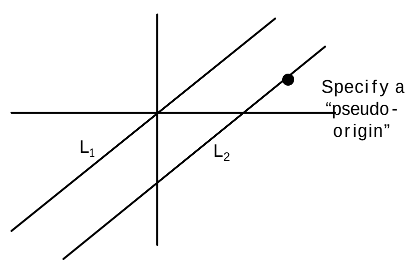
Hence no closure for linear operator: $f(\mathbf{v}) = -2\mathbf{v}$



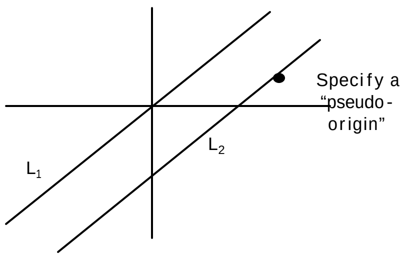
What is the problem?



Solution



Frame



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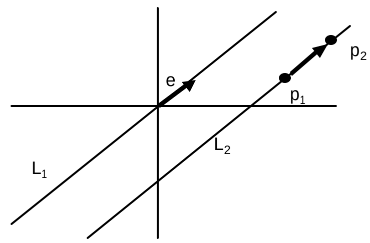
Frame Specification



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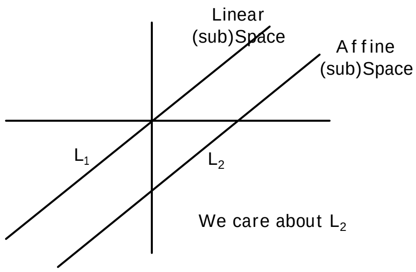
(sub)Spaces



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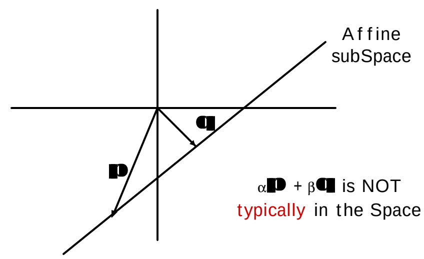
Barycenters: Let's return to the closure problem



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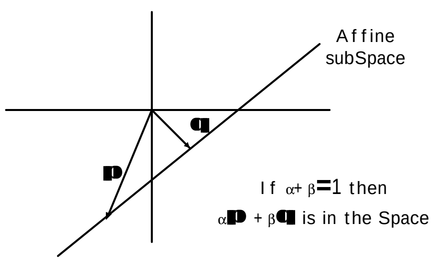
Closure revisited



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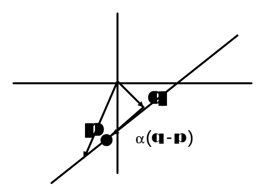
Closure revisited

- Show $\alpha p + (1-\alpha) q$ is in the space
- Note $\alpha p + (1-\alpha) q =$ $q + \alpha(q - p)$

Barycenter



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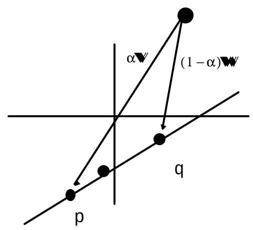
In fact:



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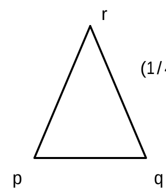
Exercise: Barycenter



What is the barycenter (1/4)p +(1/4)q +(1/2)r ?

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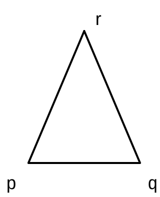
Exercise: Barycenter



What is the barycenter p + q - r ?

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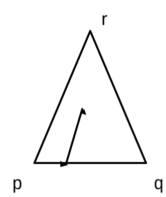
Computation of Barycenter



What is the barycenter of (1/4)p + (1/4)q +(1/2)r ?

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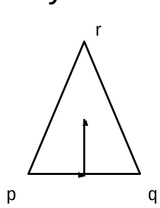
Recursive computation of Barycenter



What is the barycenter of (1/4)p + (1/4)q +(1/2)r ?

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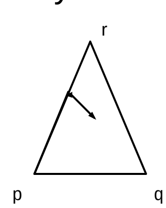
Recursive computation of Barycenter



What is the barycenter of (1/4)p + (1/4)q +(1/2)r ?

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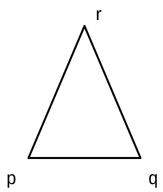
Exercise: Recursive Computation of Barycenter



What is the barycenter $p + q - r$?

Affine operator

An operator is affine if it preserves barycenters:

$$f(\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n) = \alpha_1 f(x_1) + \alpha_2 f(x_2) + \ldots + \alpha_n f(x_n)$$

Provide $\Sigma\alpha_i=1$

Next Time

- Curves
- Parametric vs. Implicit forms
- Splines: Hermitian, Bezier, B-spline
- Polar forms
- Recursive computation

Today's film clips: Alien Worlds

- 2001: Space Odyssey
- Titan A.E.
- Gas Planet

Assignment

- Maya tutorial 3&4
- Grade tutorial 1&2