CS181a: Computer Animation

B-Splines

Z Sweedyk

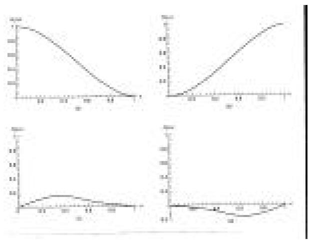
Today

- B-spline curves
  - Uniform
  - Non-uniform
- NURB curves
- Surfaces
- More Disney principles

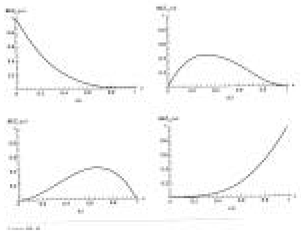
Curves

- Hermitian
- Bezier
- **B-Spline**

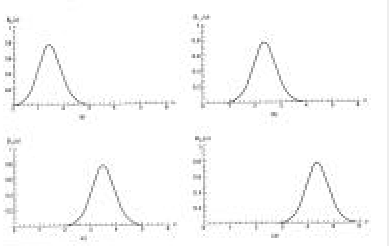
Hermitian Cubic Blending Function



Bezier Cubic Blending Functions



Some B-Spline Blending Functions



Bezier vs. B-spline

BEZIER

- Each control point affects entire curve
- Number of control points determine degree of polynomial
- Simple

B-spline

- Very localized control
- Number of control points give an upper bound on degree of polynomial
- More complicated

Bezier vs. B-Spline

Bezier is a special case of B-spline!!!!!

Exercise

http://theory.lcs.mit.edu/~boyko/classes/b-spline.html

or

http://www.doc.ic.ac.uk/~dfg/AndysSplineTutorial/BSplines.html

Bezier vs. B-spline

(n+1 control points)

Bezier: $P(t) = \Sigma_{k=0..n}\, p_k\, BZ_{k,n}(t)$ $t\in[0,1]$

$BZ_{k,n}(t) = C(n,k)t^k(1-t)^{n-k}$

B-Spline: $P(t) = \Sigma_{k=0..n} p_k B_{k,d}(t)$ $t\in[t_{min},t_{max}]$

Bezier vs. B-spline

(n+1 control points)

Bezier: $P(t) = \Sigma_{k=0..n}\, p_k\, BZ_{k,n}(t)$ $t\in[0,1]$

$BZ_{k,n}(t) = C(n,k)t^k(1-t)^{n-k}$

B-Spline: $P(t) = \Sigma_{k=0..n} p_k B_{k,d}(t)$ $t\in[t_{min},t_{max}]$

New parameter

New parameters

B-spline properties for parameters n,d

- n+1 control points are specified and n+1 blending functions used
- the curve is a polynomial of degree d-1 with C^{d-2} continuity

Uniform B-spline

- Specification
- Blending Functions
- Simple Example

Uniform B-Spline

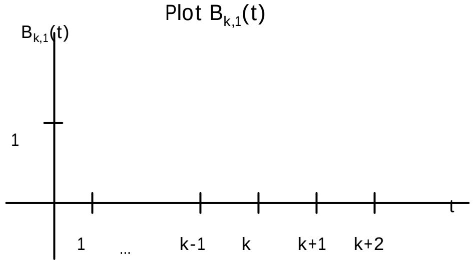
- Specification
  - n+1 control points $p_o, p_1, \ldots, p_n$
  - Degree d of curve where $1 \le d \le n$
  - Default range of t: [0,n+d+1]

Blending Functions

- Compute $B_{k,d}$ for k=0,…,n
- $B_{k,1}(t) = \begin{cases} 1 \text{ if } k \le t \le k+1 \\ 0 \text{ otherwise} \end{cases}$
- $B_{k,d}(t) = [(t-k)/(d-1)]B_{k,d-1} + [(k+d-t)/(d-1)]B_{k+1,d-1}$

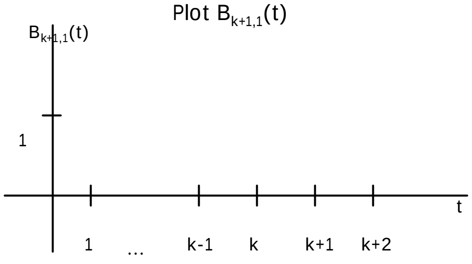
Exercise 1______________

Plot $B_{k,1}(t)$



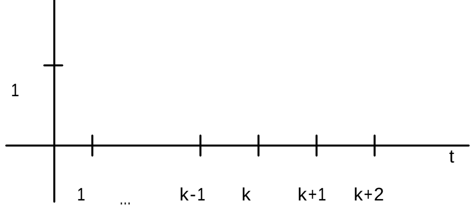
Exercise 2______________

Plot $B_{k+1,1}(t)$



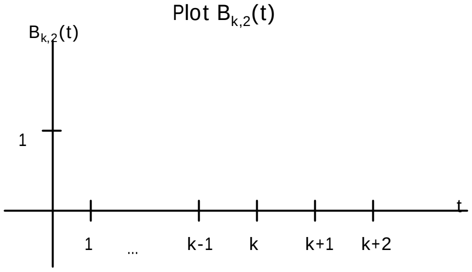
Exercise 3______________

Plot the functions t-k and k+2-t



Exercise 4_____________

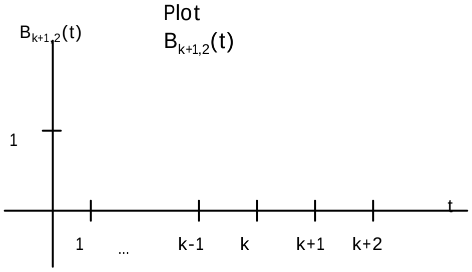
Plot $B_{k,2}(t)$



Exercise 5_____________

Plot

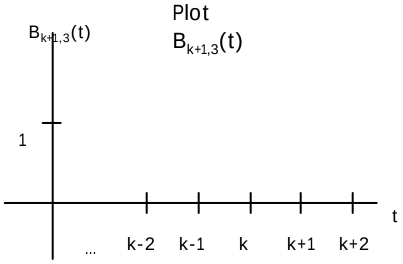
$B_{k+1,2}(t)$



Exercise 6_____________

Plot

$B_{k+1,3}(t)$



Uniform B-spline

(n+1 control points)

B-Spline: $P(t) = \Sigma_{k=0..n} p_k B_{k,d}(t)$ $t \in [t_{min}, t_{max}]$

New parameters

t_{min} = d-1

t_{max} = n+1

Exercise 7_____________

- Sketch X(t) and Y(t) for the uniform B-spline when d=2 and n=2 where
  - P_o=(1,3)
  - P_1=(-1,2)
  - P_2=(3,1)

General B-spline

(non-uniform)

- we specify intervals for the parameter t: $t \in [u_i, u_{i+1}]$, i=0,...,n+d+1, where $u_i \leq u_{i+1}$
- $(u_0, u_1, \ldots, u_{d+n-1})$ is called the knot vector
- u_i is called a knot
- uniform B-spline means $|u_{i+1} - u_i|$ is constant
- we've been using (0,1,...,n+d-1) as knot vector

B-spline properties for parameters n,d

- n+1 control points are specified and n+1 blending functions used
- the curve is a polynomial of degree d-1 with C^{d-2} continuity

B-spline properties for parameters n,d

- the blending function $B_{k,d}$ is defined over d subintervals starting at knot k
- the spline is defined for $t \in [d-1, n+1]$
- each section of spline is influenced by d control points
- each control point affects at most d sections

Nonuniform vs. Uniform

Increased flexibility with nonuniform

  - Blending functions have different shapes in different intervals
  - Knot multiplicity reduces continuity at knots

NURBS

- NonUniform Rational B-Spline

$$P(t) = \frac{F_1(t)}{F_2(t)}$$

where $F_1(t) = \Sigma_{k=0\ldots n} w_k p_k B_{k,d}(t)$

and $F_2(t) = \Sigma_{k=0\ldots n} p_k B_{k,d}(t)$

Advantages of NURBS

- Provide exact represation for conic curves: circle, ellipse, etc.
- Invariant with respect to perspective viewing

Surfaces

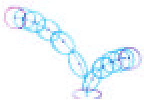
- Tensor product of curves

Disney's Principles of Animation

- Squash & Stretch
- Anticipation
- Staging
- Straight Ahead Action & Pose to Pose
- Follow Through & Overlapping Action
- Slow In & Out
- Arcs
- Secondary Action
- Timing
- Exaggeration
- Solid Drawing
- Appeal

Squash and Stretch

- Organic objects deform when they move
- Maintain volume



Overlapping Action

- Components of Organic objects move at different rates

Anticipation

- Forecast action

Films

- Mickey
- Fantasia: Sorcerer
- Stanley & Stella
- Gas Planet