CS 181b
Advanced Topics in Algorithms
Spring 2002
Problem Set 1b
Due Tuesday, January 29 in class

Please remember that all problem set submissions must be typeset, preferably in LaTeX. Please keep an electronic version of your solution set for your records.

1. **[10 Points] The Arithmetic and Geometric Mean.** This problem is a good review of the subtleties of mathematical induction. It's also a result that we'll be using in class next week!

   Let $x_1, \ldots, x_n$ be positive real numbers. The *arithmetic mean* of these numbers is defined to be $\frac{x_1+x_2+\ldots+x_n}{n}$ and the *geometric mean* is defined to be $(x_1 x_2 \cdots x_n)^{1/n}$. In this problem we show that the arithmetic mean of n numbers is at least as large as the geometric mean of those numbers.

   (a) Use induction to show that if $x_1 x_2 \cdots x_n = 1$ then $x_1 + x_2 + \ldots + x_n \geq n$. Observe that this is a special case of the statement we are trying to prove.

   (b) Use this fact to show that the arithmetic mean is at least as large as the geometric mean. (No induction required here; just a little algebra.)

2. **[10 Points] Binary Counters with Reset!** Consider the binary counter problem that we examined in class. The counter starts at 0. In addition to the INCREMENT operation, we also wish to support the RESET operation which resets the counter to 0. Show that any sequence of INCREMENT and RESET operations can be performed in $O(n)$ time. (*Hint:* Keep a pointer to the high-order 1.)

3. **[10 Points] Extendible Arrays Revisited.** In class we examined extendible arrays with operations INSERT and DELETE. In particular, our rule for DELETE was that the array gets halved when the load drops below 1/4. Now consider a different policy in which we contract the array by 2/3 when the load drops below 1/3. Use the potential function

   $$\Phi(A) = |2 \times \text{num}(T) - \text{size}(T)|$$

   to show that the amortized cost of a DELETE operation is bounded by a constant.

4. **[20 Points] Dictionaries.** Problem 17-2 from Cormen *et al.*. This problem is attached for your convenience.

5. **[25 Points] Weight-Balanced Trees!** Problem 17-3 from Cormen *et al.*. This problem is attached for your convenience.