CS 181b
Advanced Topics in Algorithms
Spring 2002
Problem Set 5a
Due Thursday, February 21 in class

1. **[15 Points] Tally Circuits!** A tally circuit has n binary inputs and $\lceil \log_2(n+1) \rceil$ outputs. Interpreted as a binary number, the outputs give the number of 1's in the input. For example, if the input is 1101111 then the output is 110, indicating that there are six 1's in the input. Describe a tally circuit with depth $O(\log n)$ and size $O(n)$. Show your analyses.

2. **[15 Points] Majority Circuits!** A majority circuit with n boolean inputs $x_1, x_2, \ldots, x_n$ outputs a 1 if $x_1 + x_2 + \ldots + x_n > n/2$ and outputs 0 otherwise. Describe a circuit for an n-input majority function that has depth $O(\lg n)$. Describe and analyze your circuit carefully. What is the size of your circuit?