**CS 181b, Advanced Algorithms**
**Spring 2002**
**Homework 8b**
**Due Tuesday, April 2**

1. [**40 Points**] **Another Version of Discrete Calculus!** In this problem we explore another version of discrete calculus that is similar, but not identical, to the version we examined in class.

   (a) [**5 Points**] **Warmin' Up!** In class, we defined $\Delta(f(x))$ to be $f(x+1)-f(x)$. Here, we'll explore a new discrete derivative $\Delta'$ defined by $\Delta'(f(x)) = f(x) - f(x-1)$.

      i. What is $\Delta'(x^2)$?

      ii. What is $\Delta'(x^{\underline{2}})$?

   (b) [**5 Points**] **Yield to the Rising Power!** Aha! That didn't turn out so great. Let's apply the standard trick of "defining our way out of trouble!" In particular, we'll define yet another type of exponentiation. Let $x^{\overline{m}}$, pronounced "$x$ to the $m$ rising", be defined by $x(x+1)(x+2)\ldots(x+m-1)$. (Notice that this is the product of $m$ consecutive terms.) What is $\Delta'(x^{\overline{m}})$? Show your work.

   (c) [**5 Points**] **Checking out the Properties of $\Delta'$.**

      i. Use the definition of $\Delta'$ to prove that $\Delta'(f(x)+h(x)) = \Delta'(f(x)) + \Delta'(h(x))$.

      ii. Show that $\Delta'(c \cdot f(x)) = c \cdot \Delta'(f(x))$.

   (d) [**10 Points**] **And now for the New Definite Summation!** Now we are compelled to define $\sum_a^b{}' f(x)\delta x$ to be $g(b) - g(a)$ where $\Delta'(g(x)) = f(x)$. Prove the Second Fundamental Theorem of Discrete Calculus:

   $$\sum_a^b{}' f(x)\delta x = \sum_{k=a+1}^{b} f(k).$$

   (e) [**5 Points**] **And now Some Applications...** Use the Second Fundamental Theorem of Discrete Calculus to find closed forms for each of the summations below. Be sure to show your work.

      i. $\sum_{k=1}^{n} k$.

      ii. $\sum_{k=1}^{n} k^2$.

   (f) [**5 Points**] **Summation by Parts!** Now expand $\Delta'(u(x)v(x))$ and derive a new summation by parts formula from it.

   (g) [**5 Points**] **Using Summation by Parts.** So isn't this slick?! Now, use the Summation by Parts formula above to find a nice closed form for

   $$\sum_{k=1}^{n} k2^k.$$

   Be sure to show each step of your work.

2. **[15 Points] The Rise and Fall of the Exponent!** In this problem we'll take one last look at the Discrete Calculus.

   (a) How should we define $x^{\overline{-m}}$? Check that your definition is "good" by making sure that $\Delta'(x^{\overline{-m}}) = -mx^{\overline{-(m+1)}}$.

   (b) Now we'll see that rising powers and falling powers are intimately related! First, show that $x^{\overline{m}} = \frac{1}{(x-1)^{\underline{-m}}}$.

   (c) Next, show that $x^{\underline{m}} = \frac{1}{(x+1)^{\overline{-m}}}$.