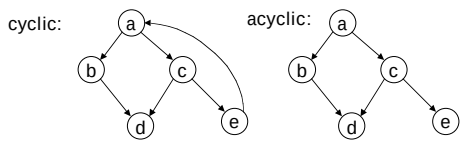
Function Decomposition

- To **solve problems** using functions, we typically:
  - Express the problem informally as a function, with input and output.
  - Break down the function as a composition of simpler functions.
  - Repeat this process, until we are using only functions that are built-in.

Function Decomposition Example (which might use anonymous functions)

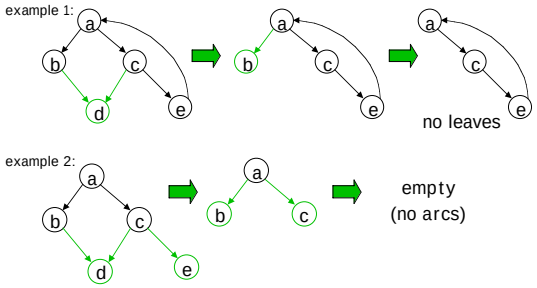
- Construct a function that will tell whether a directed graph, represented as a list of arcs, is acyclic.



"Pruning" Method

- Rosalind B. Marimont
  A new method of checking the consistency of precedence matrices
  Journal of the ACM **6**, 164-171, 1959
- Pruning away any arcs that point to leaves does not change the cyclic/acyclic nature of the graph.
- Pruning such arcs may produce additional leaves.
- Prune until no further pruning is possible:
  - If the result is empty, the original graph was acyclic.
  - If not, it was cyclic.

Examples of Pruning (leaves shown in green)

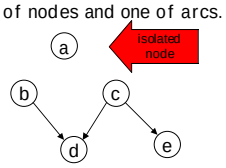


Pruning with Graphs as Lists

- Example 1:
  - [[a, b], [a, c], [b, d], [c, d], [c, e], [e, a]] ➡
  - [[a, b], [a, c], [c, e], [e, a]] ➡
  - [[a, c], [c, e], [e, a]] (no leaves)
- Example 2:
  - [[a, b], [a, c], [b, d], [c, d], [c, e]] ➡
  - [[a, b], [a, c]] ➡
  - [ ]

Note

- We are assuming that every node in the graph is on one or the other end of an arc, i.e. there are no isolated nodes, as in the graph below.
- Otherwise, we'd have to represent the graph with two lists: one of nodes and one of arcs.



Functional Code

- Basic idea:
  - As long as there is a leaf:
    Remove leaves and their attached arcs
- Translation:
- isAcyclic(Graph) =

```
null(iterate(removeLeaves, hasLeaf, Graph));
```

- null: test for empty list
- iterate: iterate first arg. as long as second arg. true
- removeLeaves: remove all leaves and their arcs
- hasLeaf: test whether there is a leaf
- Graph: starting graph

hasLeaf

- A Graph has a leaf iff isLeaf is true for one of its nodes.
- hasLeaf(Graph) =

```
some((Node)=>isLeaf(Node, Graph), nodes(Graph));
```

- some: test whether first arg. is true for some element of second arg.
- (Node)=>isLeaf(Node, Graph): true when Node is a leaf of this Graph
- nodes(Graph): list of nodes of Graph

isLeaf

- A node is a leaf if it is not the first of any arc in the graph.
- isLeaf(Node, Graph) =

```
!member(Node, map(first, Graph));
```

- !: 'not' operator
- map(first, Graph): all nodes of Graph that begin some arc

nodes(Graph)

- nodes(Graph) =

```
remove_duplicates(append(map(first, Graph),
                         map(second, Graph)));
```

- remembering our assumption: that every node in the graph is on one or the other end of an arc, i.e. there are no isolated nodes, as in the graph below.

remove_leaves

- To remove the leaves:
  remove any arc that points to a leaf
- removeLeaves(Graph) =

```
drop((Arc)=>isLeaf(second(Arc), Graph),
     Graph);
```

- Graph: the list of arcs in the graph
- second(Arc): the node to which Arc points

iterate

- iterate(action, continue, State) =

```
continue(State) ?
        iterate(action, continue, action(State))
       : State;
```

conditional expression (as in C++, Java)
P ? A : B
means if P is true then the value of the expression is A; otherwise it is B.

Improving Efficiency

- Recall the ***isAcyclic*** example.
- There there might be occasion to compute the same thing multiple times, for example
  isLeaf(Node, Graph)
  may be called **multiple times** for a given Graph:
  - hasLeaf(Graph) =
    some((Node)=>isLeaf(Node, Graph), nodes(Graph));
- **Each** time isLeaf is called, map(first, Graph) is recomputed
  - isLeaf(Node, Graph) = !member(Node, map(first, Graph));
- It may be better to compute map(first, Graph) "up front" and pass it to isLeaf.

Improving Efficiency

- Computing up front means an extra argument to isLeaf, which will may clutter the meaning of a given function:
- Below we "promote" map(first, Graph) out of isLeaf
  - hasLeaf(Graph) =
    some((Node)=>isLeaf(Node, map(first, Graph)), nodes(Graph));
  - isLeaf(Node, Firsts) = !member(Node, Firsts);
- However, it is still may be called once for each node.
- In order to avoid recomputation, we need to promote it out of the call to *some*.
- This can be done with a **local equation**, or "equational guard":
  - hasLeaf(Graph) =
    Firsts =map(first, Graph),
    some((Node)=>isLeaf(Node, Firsts), nodes(Graph));

equational guard

Improving Efficiency

- If we prefer not to use an equational guard, we can pass Firsts as an argument to hasLeaf:
- Below we "promote" map(first, Graph) out of isLeaf
  - hasLeaf2(Graph, Firsts) =
    some((Node)=>isLeaf(Node, Firsts), nodes(Graph));
- This will necessitate introduction of a new definition for the original 1-argument hasLeaf:
  - hasLeaf(Graph) = hasLeaf2(Graph, map(first, Graph));

Improving Efficiency

- Alas, we overlooked at least one little detail:
- *isLeaf* is used in removeLeaves as well as in hasLeaf, so we'll similarly have to adjust its use there.
  - removeLeaves(Graph) =
    removeLeaves2(Graph, map(first, Graph));
  - removeLeaves2(Graph, Firsts) =
    drop(((Arc)=>isLeaf(second(Arc), Firsts)), Graph);

Improving Efficiency

- There is still one obvious inefficiency:
  - map(first, Graph) is computed in both hasLeaf and removeLeaves; We'd prefer to compute it only once.
  - The low-level definition (using recursion instead of *iterate*) then might be:
    - isAcyclic(Graph) =
      Firsts = map(first, Graph),
      hasLeaf2(Graph, Firsts) ?
      isAcyclic(removeLeaves2(Graph, Firsts))
      : null(Graph);
  - Alternatively, we could construct a different version of iterate, but it would seem to be rather special purpose.

Closing Notes on Efficiency

- In previous slides, we used the property of ***referential transparency*** of functional languages (that expressions can be substituted for equivalent expressions) to improve efficiency.
- It would not generally be possible to do such substitutions in an imperative language; procedures that have side-effects cannot be substituted so freely.
- Transforming a program in the manner shown may impair its clarity and readability; so it is better to maintain a perhaps less-efficient reference version apart from the "production" version.