Search Trees & Graphs

Recursion on Trees

Tree Dichotomy

- We often want a different dichotomy than for lists: either
  - The tree has a single node (and is thus a **leaf**), or
  - The tree is a node with offspring (and is thus **not a leaf**).

Model Independence

- We can abstract away the specific representation being used:
  - **isLeaf**(T) 1 when T is leaf, 0 otherwise.
  - **offspring**(T) the list of offspring of a non-leaf, undefined otherwise
- The implementation depends on which tree model we are using.

Two Possible Implementations

- Unlabeled-Tree Implementation:
  - **makeTree**(ListSubTrees) = ListSubTrees;
  - **isLeaf**(T) = atomic(T);
  - **offspring**(T) = T;
- Labeled-Tree Implementation:
  - **makeTree**(Root, ListSubTrees) = [Root | ListSubTrees];
  - **isLeaf**(T) = null(rest(T));
  - **offspring**(T) = rest(T);
  - **root(T)** = first(T);

Recursion on Trees

- Basis: What happens on a single leaf.
- Induction step: What happens on a non-leaf.

Example: Height of a Tree

- The height of a tree is the length of the longest path from the root.
  - height(T) => isLeaf(T) ? 0;
  - height(T) => 1 + maxList(map(height, offspring(T));
- Let recursion do the work for you.

Searching a Tree

- Suppose we want to find all nodes with labels having a property P.
- Here we need to say whether the interior nodes are labeled or not.

Searching a Labeled Tree

- findInTree(P, T) =

  Root = root(T),

  foundInRest =
  mappend((S)=>findInTree(P, S), offspring(T)),

  P(Root) ? [Root | foundInRest] : foundInRest;

"Equational guard", or Local Definition

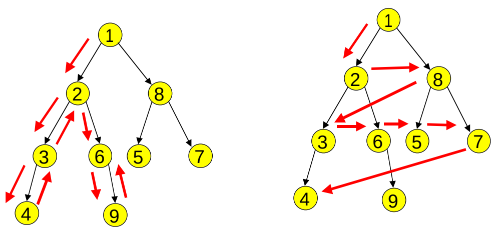
Depth-First Search

- The preceding expresses only one form of search:

  depth-first search

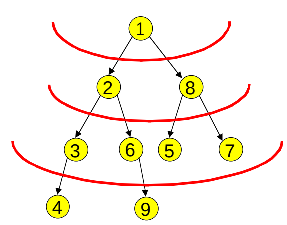
  The pattern is to search "deeper" before "broader".

Depth- vs. Breadth- First



Example: Find evens

Wavefront Analogy



Advantage of Breadth-First

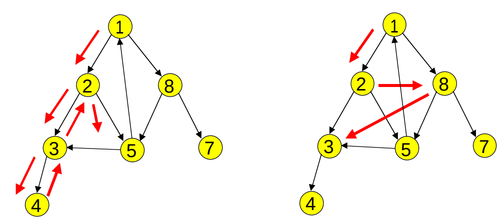
Breadth-First
Searching a Labeled Tree

- `findInTreeBF(P, T) = findInForest(P, [T]);`
-
```
findInForest(P, [ ]) => [ ];
findInForest(P, [Tree | Trees]) =>
      Root = root(Tree),
      foundInRest = findInForest(P, append(Trees, offspring(Tree)),
      P(Root) ? [Root | foundInRest] : foundInRest;
```

Searching Graphs vs. Trees

- Basic ideas still apply
- In graph, avoid re-searching same nodes due to fanin-in
  - In graph, avoid infinite loops

Depth- vs. Breadth- First in Graph



Example: Find evens

Searching Without Recursion

- Depth-First: Use Stack
- Breadth-First: Use Queue
- Avoid Fan-in and Cycles:
  - "Mark" nodes as encountered
    - Refuse to re-search from a marked node
  - Marking can be virtual, e.g. by membership on a list, or
    The node itself can be marked (non-functional programming).

Searching a Maze

- A maze is an implicit graph
- Nodes are identifiable by position
- The arcs are implicit
- Marking can be done in a "parallel array"