Harvey Mudd College
Computer Science 80
Logic for Computer Science
Spring Semester 2002

Assignment #5 – Propositional Logic: Resolution Refutation and Matching
Sample Solution

1. Consider the set of numbered clauses:

$$\overset{1}{a \vee \neg b \vee c} \qquad \overset{2}{b \vee c \vee \neg d} \qquad \overset{3}{b \vee \neg c \vee e \vee \neg f} \qquad \overset{4}{c \vee d \vee g}$$

   For each pair of clauses show all possible resolvents of the pair.

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1 & 2: One Resolvent: $a \vee c \vee \neg d$

1 & 3: Two Resolvents: $a \vee c \vee \neg c \vee e \vee \neg f$ and $a \vee b \vee \neg b \vee e \vee \neg f$

1 & 4: No Resolvents

2 & 3: One Resolvent: $b \vee \neg d \vee e \vee \neg f$

2 & 4: One Resolvent: $b \vee c \vee g$

3 & 4: One Resolvent: $b \vee e \vee \neg f \vee d \vee g$

2. Consider the set of assumptions:

   - Every fungus is either a mushroom or a toadstool.
   - Every boletus is a fungus.
   - Toadstools are poisonous, as are peach pits.
   - A boletus is not a mushroom.
   - This thing is a boletus.

   If we wish to know whether "this thing" is poisonous, we can model this with the set of formulas:

   $$\Gamma = \{f \Rightarrow (m \vee t), b \Rightarrow f, (pp \vee t) \Rightarrow p, b \Rightarrow \neg m, b\}$$

   and attempt to prove the consequence:

   $$\Gamma \models p$$

   Construct the corresponding implication, convert its negation to conjunctive normal form, and produce a resolution refutation tree showing that the consequence holds.

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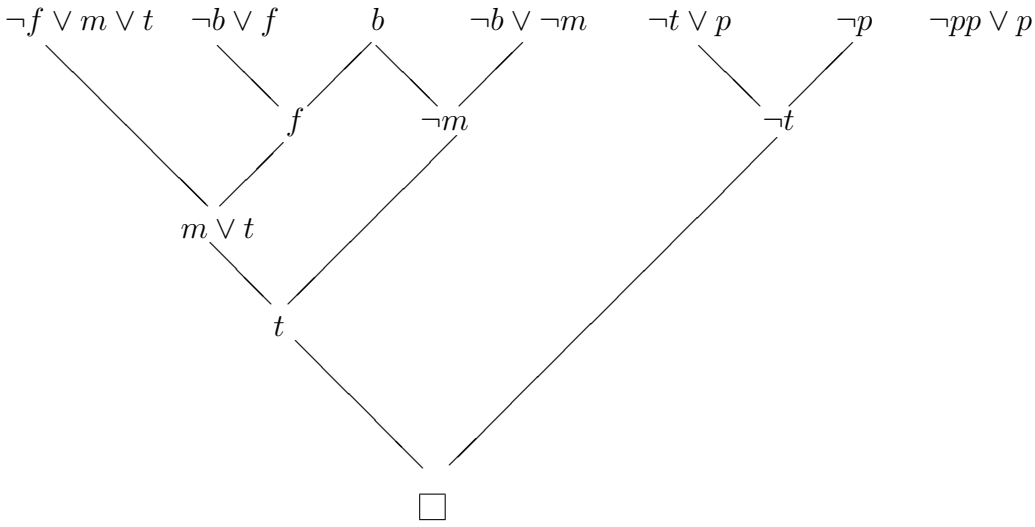
   - Construct the corresponding implication and negate it:

     $$\neg(((f \Rightarrow (m \vee t)) \wedge (b \Rightarrow f) \wedge ((pp \vee t) \Rightarrow p) \wedge (b \Rightarrow \neg m) \wedge b) \Rightarrow p)$$

   - Convert it to CNF:

     $$(\neg f \vee m \vee t) \wedge (\neg b \vee f) \wedge (\neg pp \vee p) \wedge (\neg t \vee p) \wedge (\neg b \vee \neg m) \wedge b \wedge \neg p$$

   - Produce a resolution refutation for the set of clauses:



   Note that there are many other resolvents possible at each level.

3. Consider the set of assumptions and the conclusion from problem 2.

   Construct the corresponding implication, convert it (not it's negation) to CNF, and use the "simple" validity tester to show that the original consequence holds.

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[[f, pp, t, b, Not b, p],
 [f, b, pp, t, m, Not b, p],
 [f, Not p, b, Not b, p],
 [f, b, Not p, m, Not b, p],
 [f, Not f, pp, t, b, Not b, p],
 [f, Not f, pp, t, m, Not b, p],
 [f, Not f, Not p, b, Not b, p],
 [f, Not f, Not p, m, Not b, p],
 [Not m, pp, t, b, Not b, p],
 [Not m, b, pp, t, m, Not b, p],
 [Not m, Not p, b, Not b, p],
 [Not m, b, Not p, m, Not b, p],
 [Not m, Not f, pp, t, b, Not b, p],
 [Not m, Not f, pp, t, m, Not b, p],
 [Not m, Not f, Not p, b, Not b, p],
 [Not m, Not f, Not p, m, Not b, p],
 [Not t, pp, t, b, Not b, p],
 [Not t, b, pp, t, m, Not b, p],
 [Not t, Not p, b, Not b, p],
 [Not t, b, Not p, m, Not b, p],
 [Not t, Not f, pp, t, b, Not b, p],
 [Not t, Not f, pp, t, m, Not b, p],
 [Not t, Not f, Not p, b, Not b, p],
 [Not t, Not f, Not p, m, Not b, p]]
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