

## Sets

**Definition** (Ben-Ari): A *set* is composed of *elements*.  
Notation:  $a \in S$ ,  $a$  is an element of set  $S$ , and  $a \notin S$ ,  $a$  is *not* an element of  $S$ . The set with no elements is called the *empty set*, denoted  $\emptyset$ .

- The set of traffic light colors:

$$\{\text{red, green, yellow}\}$$

- The set of strings of names of CS faculty:

$$\{\text{"Bob Keller", "Zach Dodds", "Mike Erlinger", "Josh Hodas", "Ran Libeskind-Hadas"}\}$$

- The set of CS faculty:

$$\{\text{Bob Keller, Zach Dodds, Mike Erlinger, Josh Hodas, Ran Libeskind-Hadas}\}$$

Repetition and order are meaningless in sets:

$$\{1, 2, 3, 3, 2, 1\} = \{1, 2, 3\}$$

## Sets

Sets may also be infinite:

- The set of *integers*:

$$\mathcal{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$$

- The set of *natural numbers*:

$$\{0, 1, 2, 3, \dots\}$$

Sets are often *described* rather than *enumerated*:

- The set of even integers:

$$\mathcal{EV} = \{n \mid n \in \mathcal{Z} \text{ and } n \bmod 2 = 0\}$$

- The set of prime numbers:

$$\mathcal{PN} = \{n \mid n \in \mathcal{N} \text{ and} \\ \neg \exists f. (f \in \mathcal{N} \text{ and } f > 1 \text{ and } n \bmod f = 0)\}$$

This is called *set comprehension* notation.

## Sets

Set operations:

$$\cup, \cap, \overline{A}$$

Set relations:

$$\subset, \subseteq$$

In most cases there is an implicit assumption that the members of a set belong to the same “type”.

## Ordered Sets

“If we impose an order on elements of a set, we obtain an *ordered set* which will be denoted:

$$(a_1, \dots, a_n)$$

Using parentheses rather than braces.” (Ben-Ari)

**Definition** (Ben-Ari): A finite ordered set of  $n$  elements is called an *n-tuple* or a *finite sequence*. A 2-tuple is called an [(*ordered*)] *pair*, a 3-tuple is called a *triple*. . . . An infinite ordered set is called an *infinite sequence*.

- $(red, green, yellow)$
- $(red, yellow, green)$
- $(yellow, red, yellow, red)$
- $(1, 2, 2, 3, 3, 3, \dots)$

## Cartesian Products

**Definition** (Ben-Ari): Let  $S$  and  $T$  be sets.  $S \times T$ , the *Cartesian product* of  $S$  and  $T$  is the set of all (ordered) pairs whose first element is from  $S$  and whose second element is from  $T$ .

In general, given sets  $S_1, \dots, S_n$ , the Cartesian product  $S_1 \times \dots \times S_n$  is the set of ordered  $n$ -tuples whose  $i$ -th element is in  $S_i$ .

- $\mathcal{N} \times \mathcal{N}$
- $\{\text{red, yellow, green}\} \times \{1, 2\}$

## Relations

**Definition:** Given sets  $S_1, \dots, S_n$ , an  $n$ -ary relation  $\mathcal{R}$  is a subset of  $S_1 \times \dots \times S_n$ .

The subset may be totally arbitrary:

- $\{(1, 2), (9, 12), (5, 3)\}$

Or it may be formed according to some pattern:

- $\{(1, 2), (2, 3), (3, 4), (4, 5), \dots\}$
- $\{(a, b) \mid a \in \mathcal{N} \text{ and } b \in \mathcal{N} \text{ and } b = a + 1\}$
- $\{(a, b) \mid a \in \mathcal{N} \text{ and } b \in \mathcal{N} \text{ and } a < b\}$

Properties of relations:

- Symmetry
- Reflexivity
- Transitivity