Logically Complete Sets of Connectives

Definition: A set of connectives is *logically complete* if all the other connectives can be defined in terms of connectives from the set.

In other words, Given a set $C \subset CONN$, C is logically complete if for all propositions $\Phi \in PROP$, there is a proposition $\Phi_C \in PROP$ such that Φ_C uses only connectives from C, and $\Phi \leftrightarrow \Phi_C$.

Logically Complete Sets of Connectives

Theorem: The following are logically complete sets of connectives:

- $\{\neg, \vee\}$
- $\{\neg, \wedge\}$
- $\{\neg, \Rightarrow\}$
- $\{\uparrow\}$
- $\{\downarrow\}$

Proof.

Logically Complete Sets of Connectives

Theorem (2.4.8)**:** The sets $\{\uparrow\}$ and $\{\downarrow\}$ are the only logically complete singleton sets.

Proof.

Representing Concepts in Propositional Logic

Suppose we have the following propositions:

- hj = "The jar is heated"
- bij = "The bug is in the jar"
- bd = "The bug is dead"

and we know that "If the jar is heated then if the bug is in the jar then the bug is dead." How do we represent that rule as a propositional formula?

If we know the jar is heated and that the bug is in the jar. What formula represents the idea that all these assumptions yield a dead bug?