Algorithms
Computer Science 140 & Mathematics 168
Instructor: B. Thom
Fall 2003
Homework 13a
Due on 12/12/03 (2pm, my office)

1. **[25 points] Network Reliability is NPC!**

   The Network Reliability Problem (NRP) is a famous problem arising in the area of network design. First, a few definitions. We will represent a network as an undirected graph with n vertices $v_1, \dots, v_n$. We say that two paths in the network are *disjoint* if they have no vertices in common except for the endpoints. For example, there can be two disjoint paths from vertex v_{42} to v_{100}, but v_{42} and v_{100} can be the only vertices that these paths have in common (since these vertices are the endpoints of the paths). In this definition, we're assuming that each pair of vertices considered is unique, i.e. $\forall\ v_i, v_j, i \neq j$. In other words, there are no disjoint path between vertex i and itself (the number of disjoint paths is 0). In the interest of reliability, it is desirable to have multiple disjoint paths between pairs of nodes in the network.

   The Network Reliability Problem (NRP) is defined as follows: Given is an undirected graph with n vertices $v_1, \dots, v_n$, an $n \times n$ symmetric matrix R_{ij} of positive integers, and a positive integer b. The question is whether there exists a subset S containing exactly b edges in G such that for *each pair* of vertices v_i and v_j there exist at least R_{ij} disjoint paths from v_i to v_j such that all paths are constructed using *only* edges from set S?

   One could imagine NRP being very useful when designing a specific network configuration. For example, if you had n nodes you wanted to connect together, you could begin by assuming you were going to link each node to every other (i.e. build a complete graph, $$$!), and then, given some desired minimal number of disjoint paths between node pairs, ask "How few physical links can be used to still achieve a desired disjointness?"

   Prove that NRP is NP-complete using a reduction from a problem that we have already shown to be NP-complete.