Algorithms
Computer Science 140 & Mathematics 168
Instructor: B. Thom
Fall 2003
Homework 5a
Due on 10/3/03 (Friday at 2pm)

1. [**30 Points**] **Optimizing Party Fun!**

   After your success at Snapple, you've decided to accept a job as senior algorithm designer at the well-known My-I'm-Soft Corporation. One day, the President of My-I'm-Soft, Gill Bates, comes to you with the following problem. "I'm throwing a company party," Gill says excitedly, "And I need your help! As you know, My-I'm-Soft has a hierarchical structure. You can think of it as a tree. The president, that's me, is at the root of the tree. Oh boy, I love being at the root!" You take a sip of your luke-warm diet coke (which My-I'm-Soft provides for free - what a perk!) and listen patiently as Gill continues. "Below the root are supervisors, below them are managers, below them are team leaders, etc., etc., until you get down to the leaves - the summer interns. Anyhow, to make the party fun, I thought it best that we don't invite an employee along with their immediate boss (their parent in the tree). Also, I've personally assigned every employee a real number (actually it's a double precision floating point, but never-mind that!) called their *coefficient of fun*. My objective is to invite employees so as to maximize the total sum of the coefficients of fun of all invited guests, while not inviting an employee with his or her immediate boss."

   (a) The first algorithm that Gill thought up was to simply enumerate all possible subsets of his employees, throw out those subsets that include an employee and his or her boss, find the score for each remaining subset, and finally choose the best one. There are 1000 employees at soft. My-I'm-Soft has also just purchased a Crayfish YMP that can process one trillion (10^{12}) subsets per second. How long will it take to find the optimal solution using this brute force approach on the Crayfish?

   (b) Describe in detail an efficient dynamic programming algorithm for this problem. What is the asymptotic running time of your algorithm?

   (c) Develop a greedy algorithm for the case when all coefficients of fun equal 1 and argue its correctness.