Algorithms
Computer Science 140 & Mathematics 168
Instructor: B. Thom
Fall 2003
Homework 7b
Due on 10/24/03 (Friday at 2pm)

1. [**30 Points**] **Detecting Cycles!**

   (a) Consider an **undirected graph** with V vertices and E edges. The graph may be disconnected. That is, the graph may consist of a number of separate disconnected components. Describe how depth-first search can be used to determine whether or not a graph has a cycle in $O(V + E)$ time. (That is, whether at least one of its components has a cycle.) You should assume that the graph is represented by an adjacency list. Carefully explain your algorithmic modification, why it will correctly determine when a cycle exists (if and only if it exists). Also carefully analyze its runtime, demonstrating that it has the desired asymptotic behavior. (This problem might require a bit of cleverness in the algorithm's data structures in order to get the $O(V + E)$ time!)

   (b) Does your above algorithm work if you use breadth-first search instead of depth-first search? Explain briefly.

   (c) Now show how the topological sorting algorithm can be used to determine whether a **directed graph** contains a directed cycle. Again, the graph may be disconnected. The algorithm should work in $O(V + E)$ time assuming the graph is represented as an adjacency list. Again, explain your algorithmic modification, justify its correctness, and carefully analyze its runtime.

2. [**20 Points**] **Diameters.** In this problem we will consider connected undirected graphs with no cycles. The graphs are simply represented with adjacency lists. The distance between two vertices in such a graph is just the *minimum* number of edges on a path between the vertices. The *diameter* of the graph is the *maximum* distance between all pairs of vertices. Make sure that you understand this definition well before proceeding!

   (a) Now, describe an algorithm that utilizes one of the reachability algorithms discussed in class to compute the diameter of a connected undirected acyclic graph in $O(E+V)$ time. You should justify why your algorithm is correct and carefully analyze and explain its running time.

   (b) [**10 Points Extra Credit**] Describe a Dynamic Programming algorithm that finds the diameter of a connected undirected acyclic graph in $O(E + V)$ time. Justify in a few sentences why your program works, and why it achieves the desired runtime.