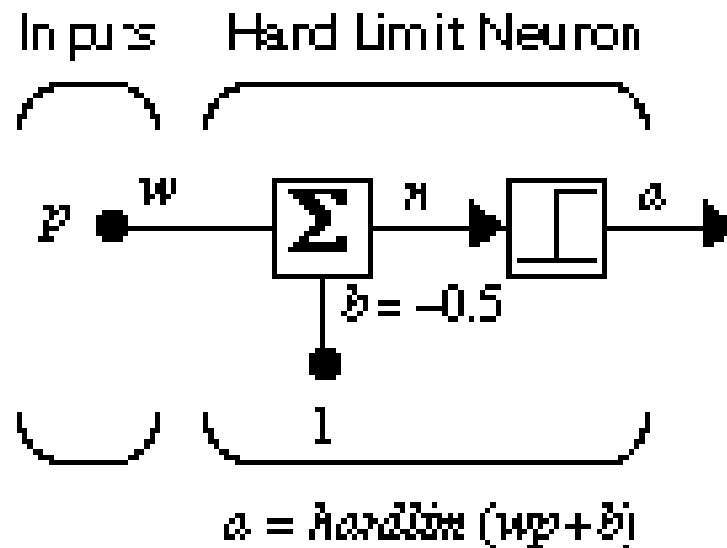

Associative Learning

(Unsupervised Hebbian and others)

Simple Associative Network

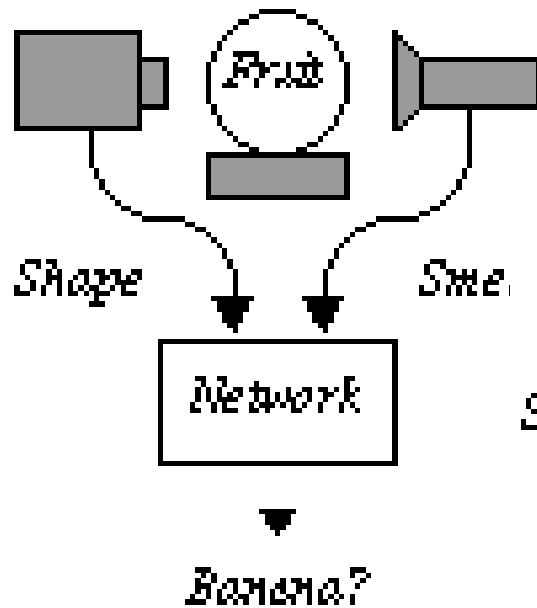


$$a = \text{hardlim}(wp + b) = \text{hardlim}(wp - 0.5)$$

$$p = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \begin{array}{l} 1, \text{ stimulus} \\ 0, \text{ no stimulus} \end{array}$$

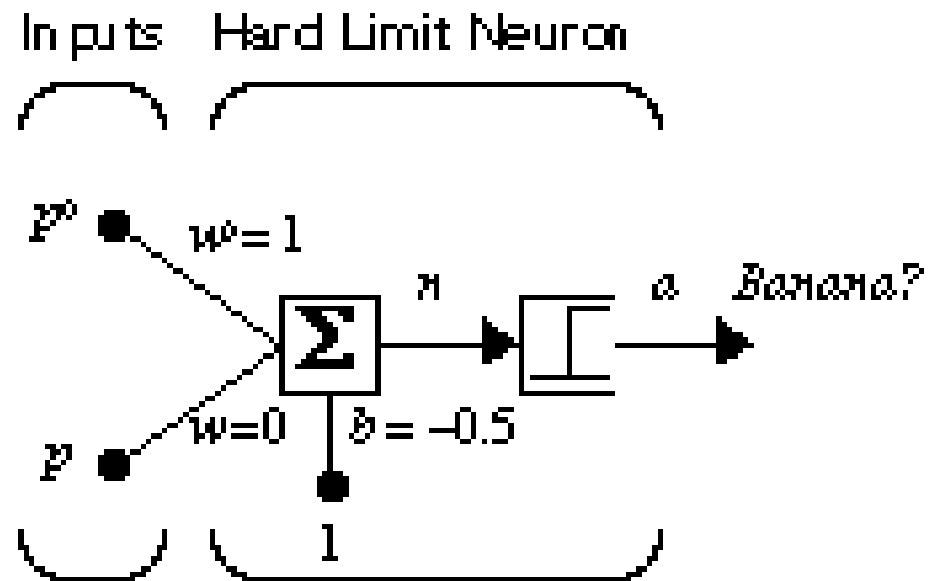
$$a = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \begin{array}{l} 1, \text{ response} \\ 0, \text{ no response} \end{array}$$

Banana Associator



Sig At of banana

Smell of banana



$$a = \text{Hardlim}(w_0 p^0 + w_1 p + b)$$

Unconditioned Stimulus

$$p^0 = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \begin{array}{l} 1, \text{ shape detected} \\ 0, \text{ shape not detected} \end{array}$$

Conditioned Stimulus

$$p = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \begin{array}{l} 1, \text{ smell detected} \\ 0, \text{ smell not detected} \end{array}$$

Unsupervised Hebb Rule

$$w_{ij}(q) = w_{ij}(q-1) + \eta a_i(q)p_j(q)$$

Vector Form:

$$\mathbf{W}(q) = \mathbf{W}(q-1) + \eta \mathbf{a}(q)\mathbf{p}^T(q)$$

Training Sequence:

$$\mathbf{p}(1), \mathbf{p}(2), \dots, \mathbf{p}(Q)$$

Banana Recognition Example

Initial Weights:

$$w^0 = 1, w(0) = 0$$

Training Sequence:

$$\{p^0(1) = 0, p(1) = 1\}, \{p^0(2) = 1, p(2) = 1\}, \square$$

$$\square = 1$$

$$w(q) = w(q-1) + a(q)p(q)$$

First Iteration (sight fails):

$$\begin{aligned} a(1) &= \text{hardlim}(w^0 p^0(1) + w(0)p(1) - 0.5) \\ &= \text{hardlim}(1 \cdot 0 + 0 \cdot 1 - 0.5) = 0 \quad (\text{no response}) \end{aligned}$$

$$w(1) = w(0) + a(1)p(1) = 0 + 0 \cdot 1 = 0$$

Example

Second Iteration (sight works):

$$\begin{aligned} a(2) &= \mathit{hardlim}(w^0 p^0(2) + w(1)p(2) - 0.5) \\ &= \mathit{hardlim}(1 \cdot 1 + 0 \cdot 1 - 0.5) = 1 \quad (\text{banana}) \end{aligned}$$

$$w(2) = w(1) + a(2)p(2) = 0 + 1 \cdot 1 = 1$$

Third Iteration (sight fails):

$$\begin{aligned} a(3) &= \mathit{hardlim}(w^0 p^0(3) + w(2)p(3) - 0.5) \\ &= \mathit{hardlim}(1 \cdot 0 + 1 \cdot 1 - 0.5) = 1 \quad (\text{banana}) \end{aligned}$$

$$w(3) = w(2) + a(3)p(3) = 1 + 1 \cdot 1 = 2$$

Banana will now be detected if either sensor works.

Problems with Hebb Rule

- Weights can become arbitrarily large
- There is no mechanism for weights to decrease

Hebb Rule with Decay

$$\mathbf{W}(q) = \mathbf{W}(q-1) + \eta \mathbf{a}(q) \mathbf{p}^T(q) - \eta \mathbf{W}(q-1)$$

$$\mathbf{W}(q) = (1 - \eta) \mathbf{W}(q-1) + \eta \mathbf{a}(q) \mathbf{p}^T(q)$$

This keeps the weight matrix from growing without bound, which can be demonstrated by setting both a_i and p_j to 1:

$$w_{ij}^{max} = (1 - \eta) w_{ij}^{max} + \eta a_i p_j$$

$$w_{ij}^{max} = (1 - \eta) w_{ij}^{max} + \eta$$

$$w_{ij}^{max} = \frac{\eta}{\eta}$$

Example: Banana Associator

$$\square = 1$$

$$\square = 0.1$$

First Iteration (sight fails):

$$\begin{aligned} a(1) &= \text{hardlim}(w^0 p^0(1) + w(0)p(1) - 0.5) \\ &= \text{hardlim}(1 \cdot 0 + 0 \cdot 1 - 0.5) = 0 \quad (\text{no response}) \end{aligned}$$

$$w(1) = w(0) + a(1)p(1) - 0.1w(0) = 0 + 0 \cdot 1 - 0.1(0) = 0$$

Second Iteration (sight works):

$$\begin{aligned} a(2) &= \text{hardlim}(w^0 p^0(2) + w(1)p(2) - 0.5) \\ &= \text{hardlim}(1 \cdot 1 + 0 \cdot 1 - 0.5) = 1 \quad (\text{banana}) \end{aligned}$$

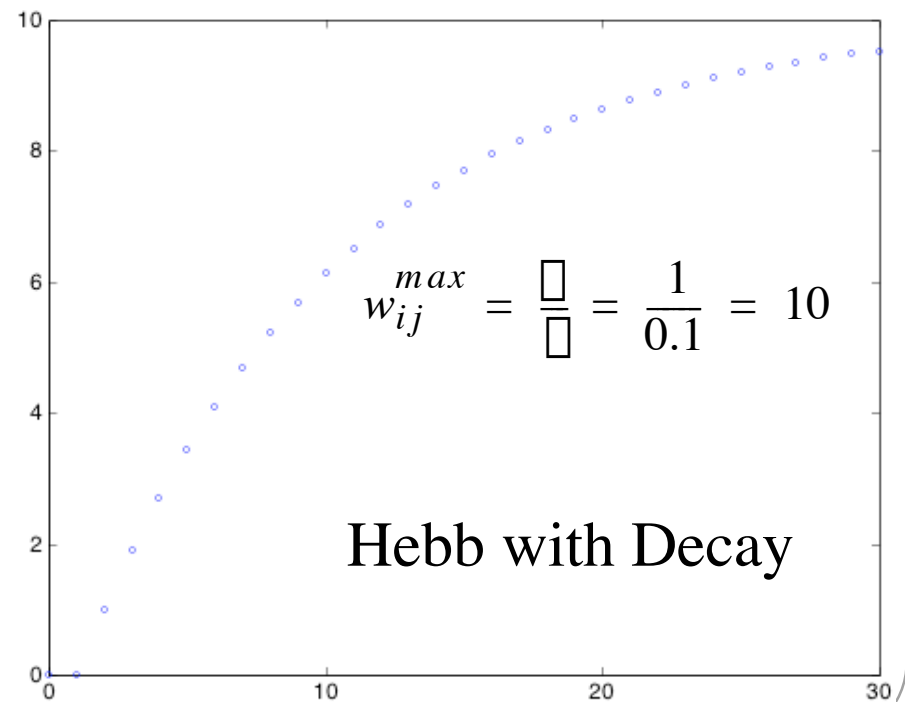
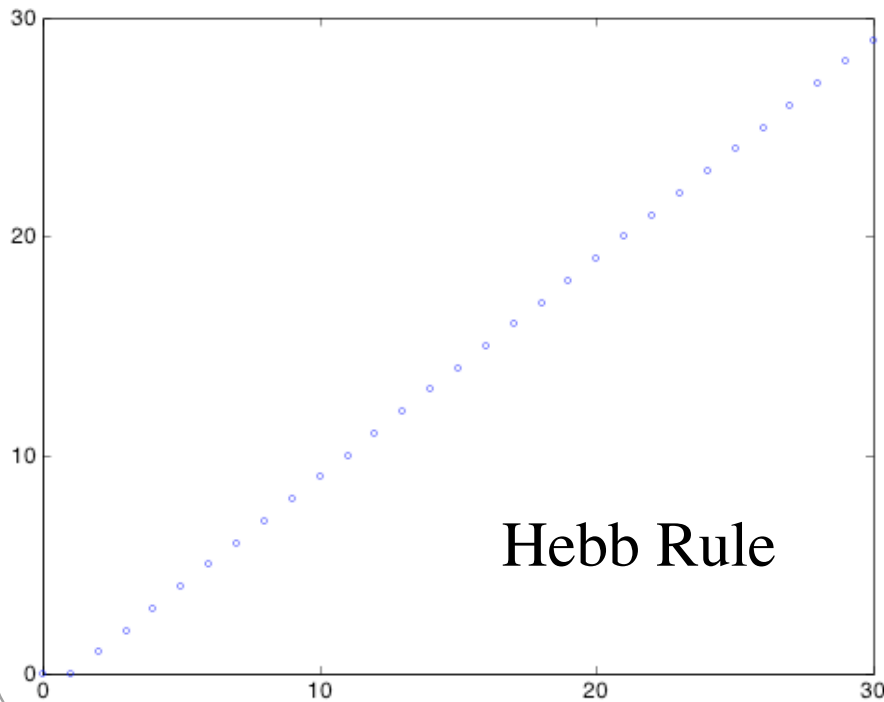
$$w(2) = w(1) + a(2)p(2) - 0.1w(1) = 0 + 1 \cdot 1 - 0.1(0) = 1$$

Example

Third Iteration (sight fails):

$$\begin{aligned} a(3) &= \text{hardlim}(w^0 p^0(3) + w(2)p(3) - 0.5) \\ &= \text{hardlim}(1 \cdot 0 + 1 \cdot 1 - 0.5) = 1 \quad (\text{banana}) \end{aligned}$$

$$w(3) = w(2) + a(3)p(3) - 0.1w(3) = 1 + 1 \cdot 1 - 0.1(1) = 1.9$$



Problem of Hebb with Decay

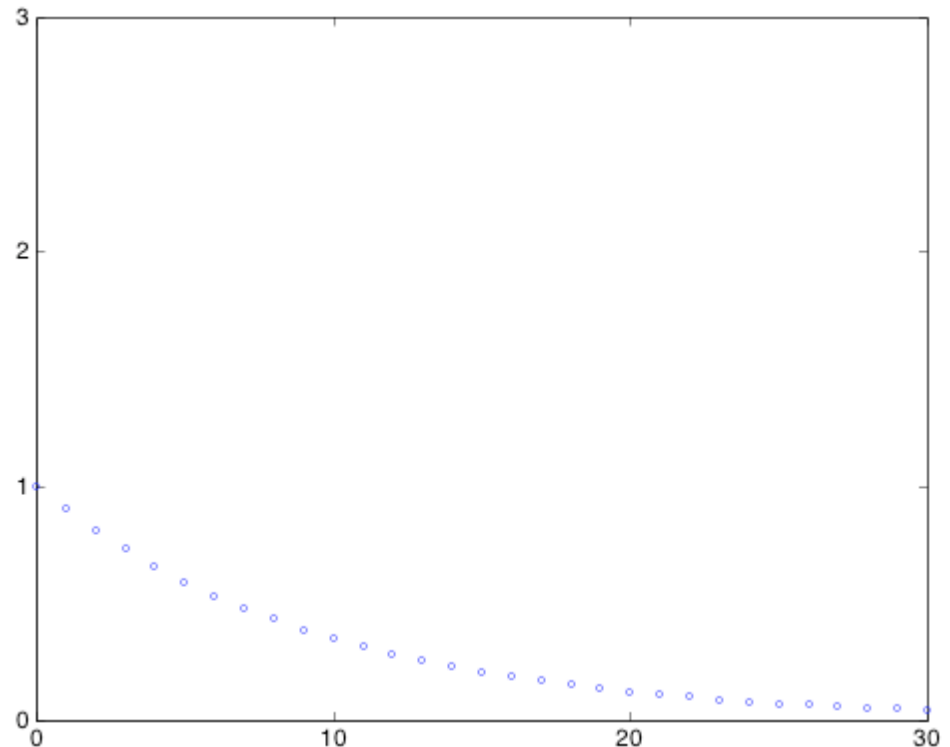
- Associations will decay away if stimuli are not occasionally presented.

If $a_i = 0$, then

$$w_{ij}(q) = (1 - \square)w_{ij}(q - 1)$$

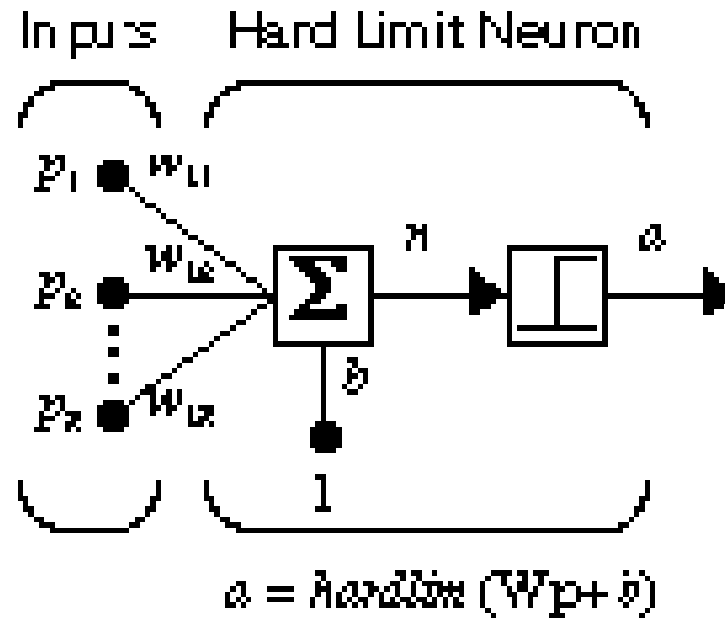
If $\square = 0$, this becomes

$$w_{ij}(q) = (0.9)w_{ij}(q - 1)$$



Therefore the weight decays by 10% at each iteration where there is no stimulus.

Instar (Recognition Network)



Instar Operation

$$a = \text{hardlim}(\mathbf{W}\mathbf{p} + b) = \text{hardlim}({}_1\mathbf{w}^T\mathbf{p} + b)$$

The instar will be active when

$${}_1\mathbf{w}^T\mathbf{p} \geq -b$$

or

$${}_1\mathbf{w}^T\mathbf{p} = \|{}_1\mathbf{w}\| \|\mathbf{p}\| \cos \theta \geq -b$$

For normalized vectors, the largest inner product occurs when the angle between the weight vector and the input vector is zero -- the input vector is equal to the weight vector.

The rows of a weight matrix represent patterns to be recognized.

Vector Recognition

If we set

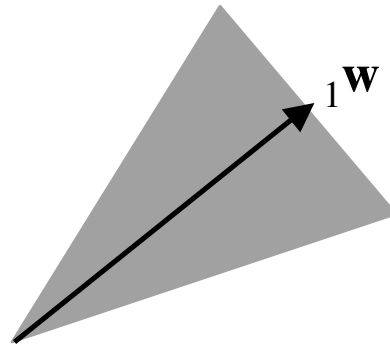
$$b = -\|\mathbf{w}_1\| \|\mathbf{p}\|$$

the instar will only be active when $\theta = 0$.

If we set

$$b > -\|\mathbf{w}_1\| \|\mathbf{p}\|$$

the instar will be active for a range of angles.



As b is increased, the more patterns there will be (over a wider range of θ) which will activate the instar.

Instar Rule

Hebb with Decay

$$w_{ij}(q) = w_{ij}(q-1) + \eta a_i(q)p_j(q)$$

Modify so that learning and forgetting will only occur when the neuron is active - Instar Rule:

$$w_{ij}(q) = w_{ij}(q-1) + \eta a_i(q)p_j(q) - \eta a_i(q)w_{ij}(q-1)$$

or

$$w_{ij}(q) = w_{ij}(q-1) + \eta a_i(q)(p_j(q) - w_{ij}(q-1))$$

Vector Form:

$${}_i\mathbf{w}(q) = {}_i\mathbf{w}(q-1) + \eta a_i(q)(\mathbf{p}(q) - {}_i\mathbf{w}(q-1))$$

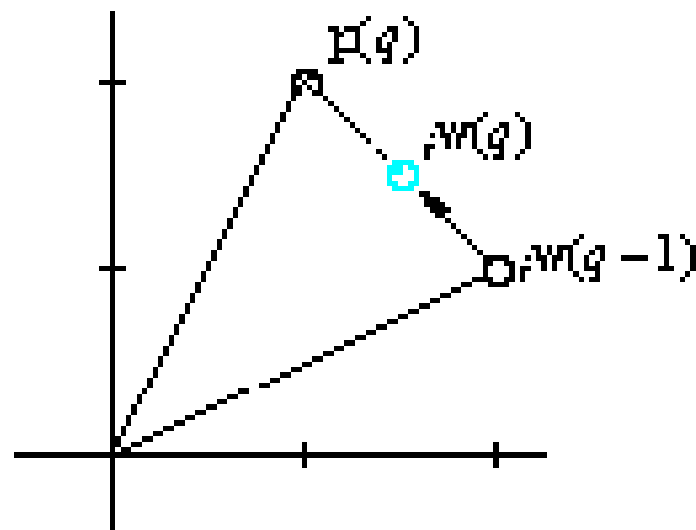
Graphical Representation

For the case where the instar is active ($a_i = 1$):

$${}_i\mathbf{w}(q) = {}_i\mathbf{w}(q-1) + \alpha(\mathbf{p}(q) - {}_i\mathbf{w}(q-1))$$

or

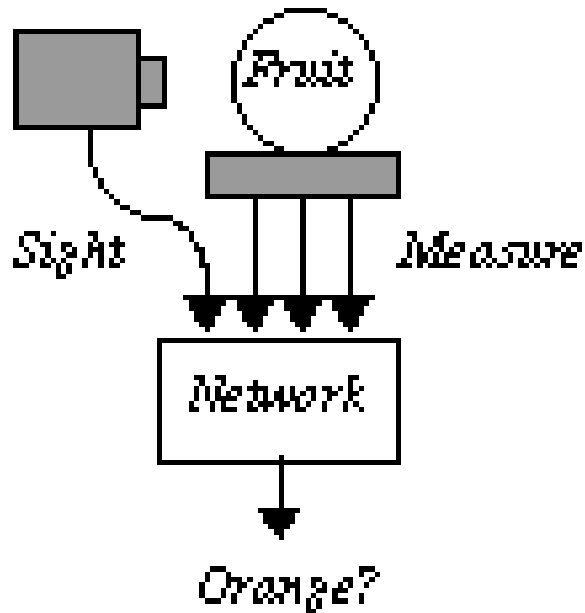
$${}_i\mathbf{w}(q) = (1 - \alpha){}_i\mathbf{w}(q-1) + \alpha\mathbf{p}(q)$$



For the case where the instar is inactive ($a_i = 0$):

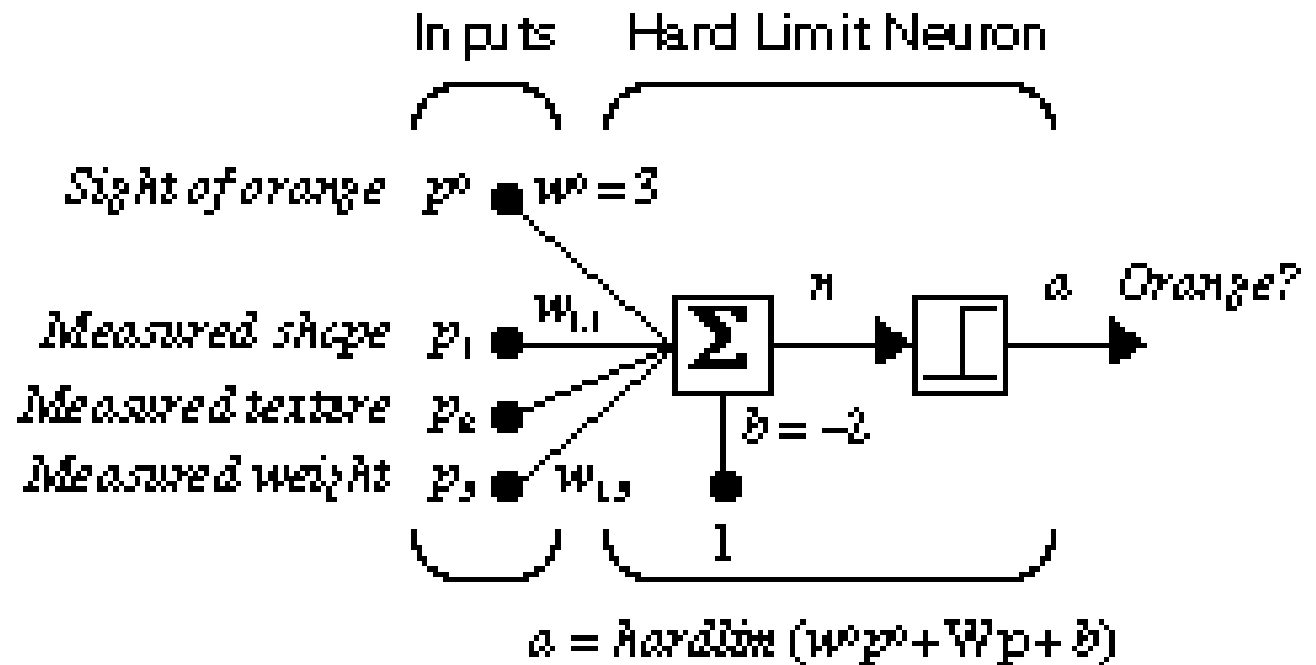
$${}_i\mathbf{w}(q) = {}_i\mathbf{w}(q-1)$$

Example



$$p^0 = \begin{cases} 1, & \text{orange detected visually} \\ 0, & \text{orange not detected} \end{cases}$$

$$\mathbf{p} = \begin{bmatrix} \text{shape} \\ \text{texture} \\ \text{weight} \end{bmatrix}$$



Training

$$\mathbf{W}(0) = {}_1\mathbf{w}^T(0) = [0 \ 0 \ 0]$$

$$\begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} p^0(1) = 0, \mathbf{p}(1) = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}, p^0(2) = 1, \mathbf{p}(2) = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}, \square$$

First Iteration ($\square=1$):

$$a(1) = \mathit{hardlim}(w^0 p^0(1) + \mathbf{W}\mathbf{p}(1) - 2)$$

$$a(1) = \mathit{hardlim} \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} \cdot 0 + [0 \ 0 \ 0] \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} - 2 \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} = 0 \quad (\text{no response})$$

$${}_1\mathbf{w}(1) = {}_1\mathbf{w}(0) + a(1)(\mathbf{p}(1) - {}_1\mathbf{w}(0)) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Further Training

$$a(2) = \text{hardlim}(w^0 p^0(2) + \mathbf{W}\mathbf{p}(2) - 2) = \text{hardlim} \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} 3 \cdot 1 + \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} - 2 \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} = 1 \quad (\text{orange})$$

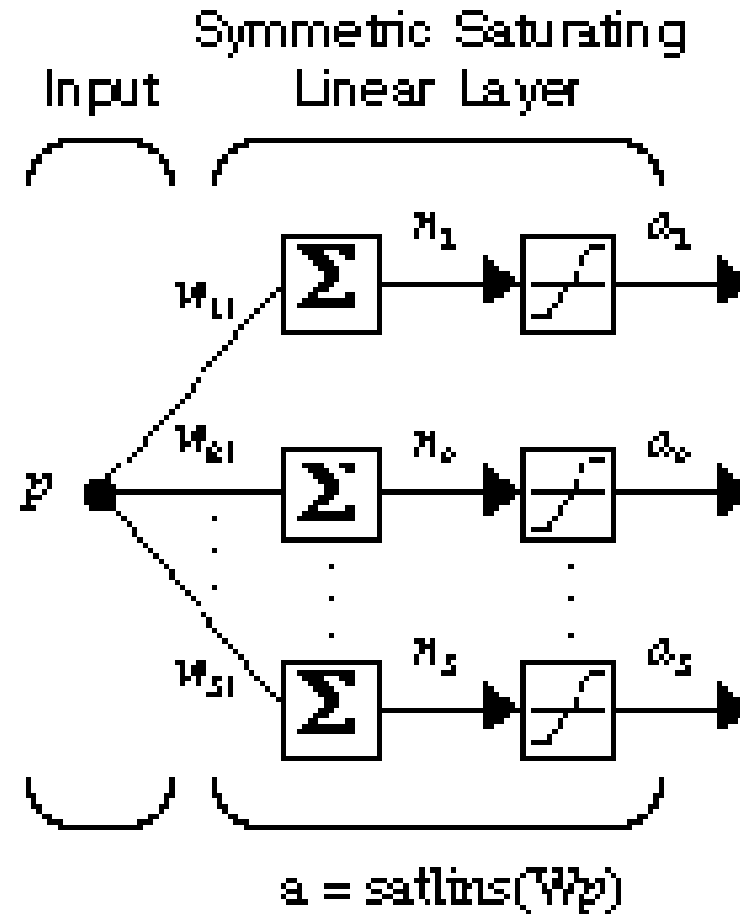
$${}_1\mathbf{w}(2) = {}_1\mathbf{w}(1) + a(2)(\mathbf{p}(2) - {}_1\mathbf{w}(1)) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$a(3) = \text{hardlim}(w^0 p^0(3) + \mathbf{W}\mathbf{p}(3) - 2) = \text{hardlim} \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} 3 \cdot 0 + \begin{bmatrix} 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} - 2 \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} = 1 \quad (\text{orange})$$

$${}_1\mathbf{w}(3) = {}_1\mathbf{w}(2) + a(3)(\mathbf{p}(3) - {}_1\mathbf{w}(2)) = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} + 1 \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

Orange will now be detected if either set of sensors works.

Outstar (Recall Network)



Outstar Operation

Suppose we want the outstar to recall a certain pattern \mathbf{a}^* whenever the input $p=1$ is presented to the network. Let

$$\mathbf{W} = \mathbf{a}^*$$

Then, when $p=1$

$$\mathbf{a} = \text{satlins}(\mathbf{W}p) = \text{satlins}(\mathbf{a}^* \cdot 1) = \mathbf{a}^*$$

and the pattern is correctly recalled.

The columns of a weight matrix represent patterns to be recalled.

Outstar Rule

For the instar rule we made the weight decay term of the Hebb rule proportional to the output of the network. For the outstar rule we make the weight decay term proportional to the input of the network.

$$w_{ij}(q) = w_{ij}(q-1) + \eta a_i(q)p_j(q) - \eta p_j(q)w_{ij}(q-1)$$

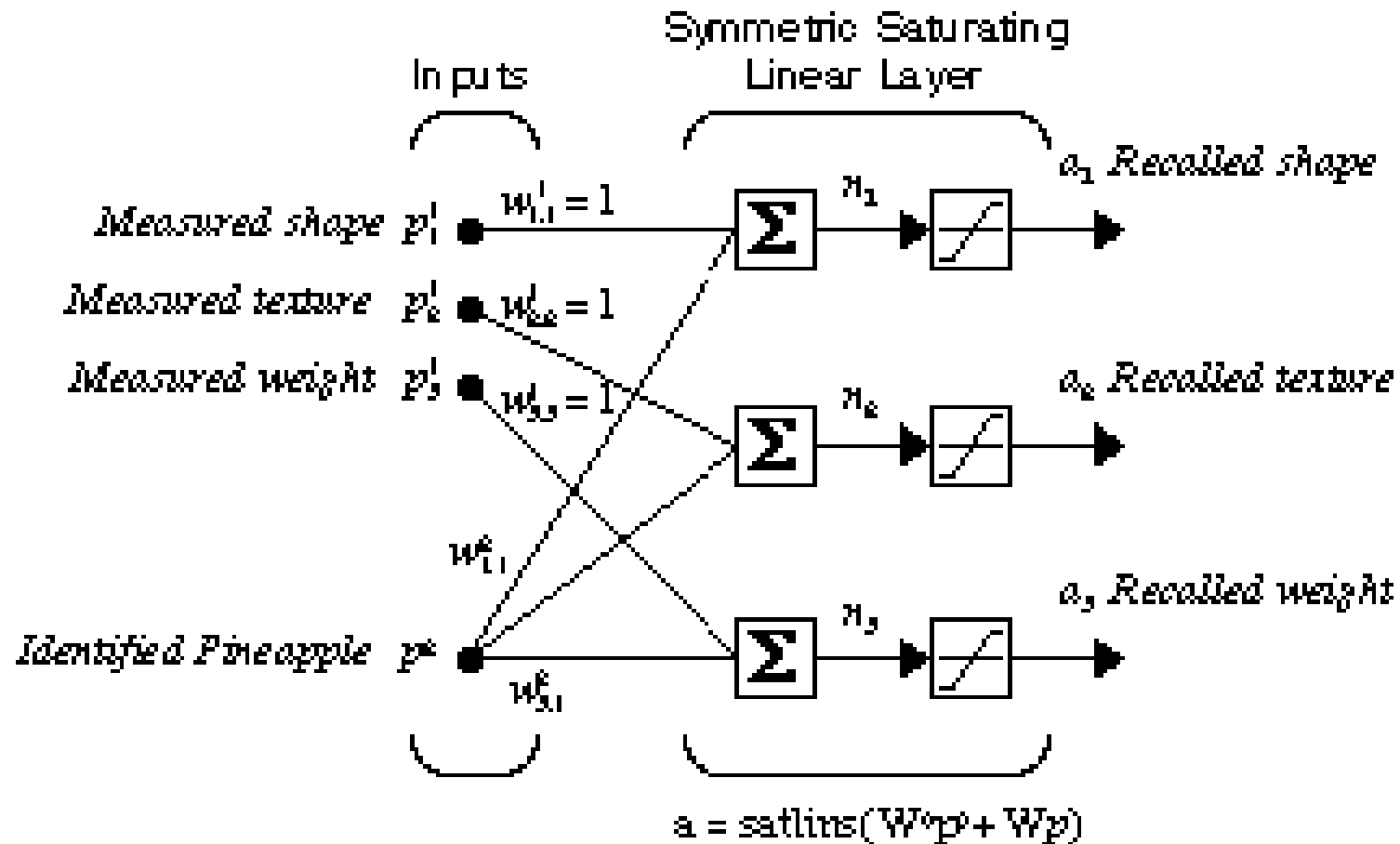
If we make the decay rate η equal to the learning rate η ,

$$w_{ij}(q) = w_{ij}(q-1) + \eta (a_i(q) - w_{ij}(q-1))p_j(q)$$

Vector Form:

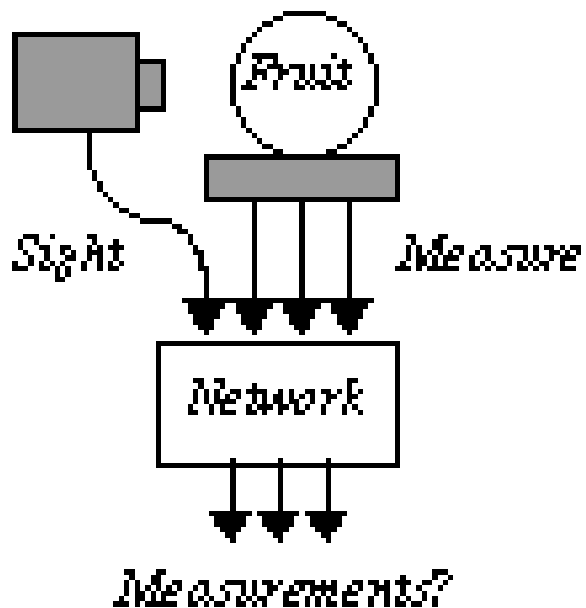
$$\mathbf{w}_j(q) = \mathbf{w}_j(q-1) + \eta (\mathbf{a}(q) - \mathbf{w}_j(q-1))p_j(q)$$

Example - Pineapple Recall



Definitions

$$\mathbf{a} = \text{satins}(\mathbf{W}^0 \mathbf{p}^0 + \mathbf{W}p)$$



$$\mathbf{W}^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{p}^0 = \begin{bmatrix} \textit{shape} \\ \textit{texture} \\ \textit{weight} \end{bmatrix}$$

$$\mathbf{p}^{\textit{pineapple}} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$p = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \begin{array}{l} 1, \text{ if a pineapple can be seen} \\ 0, \text{ otherwise} \end{array}$$

Iteration 1

$$\begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} \mathbf{p}^0(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, p(1) = 1, \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} \mathbf{p}^0(2) = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, p(2) = 1, \square$$

$$\square = 1$$

$$\mathbf{a}(1) = \mathbf{sat}(\mathbf{li} \mathbf{ns}) \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} 1 \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (\text{no response})$$

$$\mathbf{w}_1(1) = \mathbf{w}_1(0) + (\mathbf{a}(1) - \mathbf{w}_1(0)) p(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} 1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Convergence

$$\mathbf{a}(2) = \text{satlins} \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} 1 \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad (\text{measurements given})$$

$$\mathbf{w}_1(2) = \mathbf{w}_1(1) + (\mathbf{a}(2) - \mathbf{w}_1(1))p(2) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} 1 \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$\mathbf{a}(3) = \text{satlins} \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} 1 \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad (\text{measurements recalled})$$

$$\mathbf{w}_1(3) = \mathbf{w}_1(2) + (\mathbf{a}(3) - \mathbf{w}_1(2))p(2) = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} 1 \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$