Evolutionary Computation

Some References

Zbigniew Michalewicz
Genetic Algorithms + Data Structures = Evolution Programs
3rd edition, Springer-Verlag, 1993

David Goldberg
Evolutionary Computation

- Evolutionary algorithms
- Genetic algorithms
- Genetic programming
- Artificial life ("alife")

Historical

- Looked at derivation of
  - finite-state machines
  - controllers
  - data reduction
  - through successive mutations
Historical

- 1975, John H. Holland
  *Adaptation in natural and artificial systems*, MIT Press
  - Focus was on natural systems, simulation
  - Introduced current genetic algorithm idea
  - Mostly theory, some applications to:
    - game-playing
    - search programs

Among Mr. Holland’s Opuses


- See [http://www.pscs.umich.edu/jhhfest/jhh-pub.html](http://www.pscs.umich.edu/jhhfest/jhh-pub.html) for more.
Mr. Holland’s Progeny

- David Goldberg
- Stephanie Forrest
- Kenneth DeJong
- Melanie Mitchell
- John Koza
- many others
- See [http://www.pscs.umich.edu/jhffest/schedule-closed.html](http://www.pscs.umich.edu/jhffest/schedule-closed.html) for Festschrift papers.

Today
Genetic Algorithms

- An approach to difficult optimization problems (TSP, etc.)
- Heuristic, not guaranteed to find true optima
- Finds good approximations fast

GA Applications

- Hundreds, if not thousands
- All manner of optimization problems in engineering, science, finance, etc.
- Application to neural networks:
  - Evolve the structure and/or weights of a neural network, rather than train it.
Principles of Natural Selection

- Concentrate on population rather than a single individual
- Individuals that are fit enough to survive will reproduce
- Create new individuals from existing ones
  - Crossover
  - Mutation

Genetic Flow Diagram

Population

Selection criterion

Subset of Population

Operators

until an individual exists satisfying performance criterion or resources are exhausted
### Common Operators

- **Copy** (aka Reproduce): An individual from the current generation is copied into the next generation.
- **Crossover**: Two (or more) individuals from the current generation are used to form an individual in the next generation.
- **Mutate**: A single individual from the current generation is mutated to form an individual in the next generation.

### Individuals

- Individuals are represented by their genome, or **genotype**.
- The genotype may be the “program” for the actual individual, or **phenotype**.
Consider the “subset sum” problem:
- Given a set of integers S and a target value T, find a subset of S that the maximum sum \textit{without exceeding} T.

This is an optimization problem. The related decision problem, find whether there is a subset summing to \textit{exactly} T, is known to be NP-complete.

A related problem arises in cryptography.
Genetic Approach

- As the genome, use a bit-vector.

- There is one bit for each element in the set S.

- The bit is 1 iff the element is in the subset.

Example

- S = \{19, 23, 35, 52, 61, 68, 76, 84, 92\}
- T = 200
- Genome is an element of \{0, 1\}^9
- Possible individuals:
  010001001, \sum\{23, 68, 92\} = 183
  101000010, \sum\{19, 35, 84\} = 138
What can we try to produce more fit individuals?

- **Mutation**: change a random bit:
  101000010, \( \text{sum}\{19, 35, 84\} = 138 \)
  
  \[ \Downarrow \]
  111100010, \( \text{sum}\{19, 23, 35, 84\} = 161 \)

Fatal Mutations

- Note that a mutation could be “fatal”, resulting in a totally unfit individual. The “carcass” of this individual could still be in the next generation, however.
What can we try to produce more fit individuals?

- **Crossover**: Combine two individuals
  - 001110000, $\text{sum}\{35, 52, 61\} = 148$
  - 10000110, $\text{sum}\{19, 76, 84\} = 179$

  crossover point selected at random

- New genomes:
  - 001000110, $\text{sum}\{35, 76, 84\} = 195$
    - better
  - 100110000, $\text{sum}\{19, 52, 61\} = 132$

---

Crossover Variations

- Sometimes two crossover points are chosen, rather than one, and the subsequences between them are swapped.

- Just as with mutation, crossover could produce one or more individuals that are totally unfit.
Sample GA Program

- The program /cs/cs152/ga/subsum/subsum.java carries out the genetic algorithm on this problem.

- Examples:
  - go ss1.in
  - go ss2.in

Main Loop of the Subset Sum GA Program (1)

```java
public void evolve(int generations) {
    for (generation = 0; generation < generations; generation++) {
        retain(); // retain the more fit individuals
        crossover(); // perform crossover on those retained
        mutate(); // mutate the resulting population
        sort(); // sort by fitness
    }
}
```
Sample Run of subsum (1)

Subset sum problem
generations = 100
population size = 10
retain size = 5
immutable = 5
mutation rate = 0.1

target = 200.0
values = (19 23 35 52 61 68 76 84 92)

generation 0, average fitness = 33.7:
181/200 (r 19) 000111000 52 61 68
164/200 (r 36) 100010010 19 61 84
-1/200 (r 201) 000011110 61 68 76 84
-1/200 (r 201) 010111000 23 52 61 68
-1/200 (r 201) 000010110 61 76 84
-1/200 (r 201) 001111100 52 61 68 76
-1/200 (r 201) 101101101 19 35 52 68 76 92
-1/200 (r 201) 000011110 61 68 76 84
-1/200 (r 201) 101101101 19 35 52 68 76 92
-1/200 (r 201) 001100101 35 52 76 92

only two fit individuals
Sample Run of subsum (2)

<table>
<thead>
<tr>
<th>Generation</th>
<th>Average Fitness</th>
<th>Individuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64.4</td>
<td>181/200 (r 19) 000111000 52 61 68</td>
</tr>
<tr>
<td></td>
<td></td>
<td>164/200 (r 36) 100010010 19 61 84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160/200 (r 40) 000000110 76 84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>145/200 (r 55) 000010010 61 84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1/200 (r 201) 001100101 35 52 76 92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1/200 (r 201) 000011110 61 68 76 84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1/200 (r 201) 100011110 19 61 68 76 84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1/200 (r 201) 000010110 61 76 84</td>
</tr>
</tbody>
</table>

Sample Run of subsum (3)

<table>
<thead>
<tr>
<th>Generation</th>
<th>Average Fitness</th>
<th>Individuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>114.3</td>
<td>197/200 (r 3) 000110010 52 61 84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>181/200 (r 19) 000111000 52 61 68</td>
</tr>
<tr>
<td></td>
<td></td>
<td>164/200 (r 36) 100010010 19 61 84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160/200 (r 40) 000000110 76 84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>148/200 (r 52) 100011000 19 61 68</td>
</tr>
<tr>
<td></td>
<td></td>
<td>145/200 (r 55) 000010010 61 84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>96/200 (r 104) 001010000 35 61</td>
</tr>
<tr>
<td></td>
<td></td>
<td>54/200 (r 146) 101000000 19 35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1/200 (r 201) 000110011 52 61 84 92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1/200 (r 201) 001100101 35 52 76 92</td>
</tr>
</tbody>
</table>
Sample Run of subsum (4)

generation 5, average fitness = 157.7:
200/200 (r 0) 100111000 19 52 61 68
197/200 (r 3) 000110010 52 61 84
197/200 (r 3) 000110010 52 61 84
181/200 (r 19) 000111000 52 61 68
181/200 (r 19) 000111000 52 61 68
179/200 (r 21) 00110001 35 52 92
164/200 (r 36) 100010010 19 61 84
164/200 (r 36) 100010010 19 61 84
115/200 (r 85) 101010000 19 35 61
-1/200 (r 201) 000111010 52 61 68 84
best possible fit reached in generation 5

Complexity Considerations

- All possible subsets of n values can be enumerated in $2^n$ steps.
- For low n (say in the 20’s or less), this might be feasible.
- For larger n, it is not ($2^{32} = \text{about 5 billion}$).
- For large n, the genetic algorithm can produce good, if not optimal, answers in much less time than enumeration.
Other Genomes

- A bit vectors is not the only way, or necessarily the best way, to represent a genome.
- Other possibilities:
  - A list or matrix of integers or floats

Challenges in representing Genomes: Traveling Salesperson Problem

- Not every optimization problem has a genome encoding that will allow naïve mutations and crossovers.
- Consider the TSP:
  - An instinctive way to represent a genome is as a permutation of the cities on a tour.
Crossing two Permutations

- $[1, 3, 2, 6, 5, 4]$
- $[2, 3, 4, 1, 6, 5]$

Result of naïve crossing:
- $[1, 3, 4, 1, 6, 5]$
- $[2, 3, 2, 6, 5, 4]$

Unfortunately, these sequences are not permutations.

Reinterpreting permutation crossings

- $[1, 3, 2, 6, 5, 4]$
- $[2, 3, 4, 1, 6, 5]$

Interpret as:
- Insert 2,6,5 in the second genome at the first crossover point and remove those elements from wherever they occurred in the second genome.
Reinterpreting permutation crossings

- [1, 3, 2, 6, 5, 4]
- [2, 3, 4, 1, 6, 5]

Crossover points selected at random

- [1, 3, 2, 6, 5, 4]
- [2, 3, 4, 1, 6, 5]

Insert [3, 2, 6, 5, 4, 1]
Remove

Similarly, to produce the second new genome

- [1, 3, 2, 6, 5, 4]
- [2, 3, 4, 1, 6, 5]

Remove

- [1, 3, 2, 6, 5, 4]
- [2, 3, 4, 1, 6, 5]

Insert [3, 4, 1, 6, 2, 5]
Net effect of Crossover

- [1, 3, 2, 6, 5, 4]
- [2, 3, 4, 1, 6, 5]

- [3, 4, 1, 6, 2, 5]
- [3, 2, 6, 5, 4, 1]

- The results share some of the structure of both parents, which is desirable.

Keller’s TSP GA

- /cs/cs152/ga/tsp
- Uses same overall loop as the subset sum algorithm.
- The genome is now a permutation vector.
- Crossover is as described.
- Mutation consists of swapping two random elements of the permutation.
TSP GA in operation (1)

Traveling Salesperson Problem

generations = 1000
population size = 50
retain size = 25
immutable = 15
mutation rate = 0.1

TSP GA in operation (2)

Costs:

<table>
<thead>
<tr>
<th>0.0</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
<th>9.0</th>
<th>8.0</th>
<th>3.0</th>
<th>2.0</th>
<th>1.0</th>
<th>5.0</th>
<th>7.0</th>
<th>1.0</th>
<th>2.0</th>
<th>9.0</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.0</td>
<td>5.0</td>
<td>3.0</td>
<td>7.0</td>
<td>2.0</td>
<td>5.0</td>
<td>1.0</td>
<td>3.0</td>
<td>4.0</td>
<td>6.0</td>
<td>6.0</td>
<td>6.0</td>
<td>1.0</td>
<td>9.0</td>
</tr>
<tr>
<td>2.0</td>
<td>5.0</td>
<td>0.0</td>
<td>6.0</td>
<td>1.0</td>
<td>4.0</td>
<td>7.0</td>
<td>7.0</td>
<td>1.0</td>
<td>6.0</td>
<td>5.0</td>
<td>9.0</td>
<td>1.0</td>
<td>3.0</td>
<td>4.0</td>
</tr>
<tr>
<td>4.0</td>
<td>3.0</td>
<td>6.0</td>
<td>0.0</td>
<td>5.0</td>
<td>2.0</td>
<td>1.0</td>
<td>6.0</td>
<td>5.0</td>
<td>4.0</td>
<td>2.0</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>9.0</td>
<td>7.0</td>
<td>1.0</td>
<td>5.0</td>
<td>0.0</td>
<td>9.0</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
<td>3.0</td>
<td>6.0</td>
<td>8.0</td>
<td>2.0</td>
<td>5.0</td>
</tr>
<tr>
<td>8.0</td>
<td>2.0</td>
<td>4.0</td>
<td>2.0</td>
<td>9.0</td>
<td>0.0</td>
<td>3.0</td>
<td>5.0</td>
<td>4.0</td>
<td>7.0</td>
<td>8.0</td>
<td>3.0</td>
<td>1.0</td>
<td>2.0</td>
<td>5.0</td>
</tr>
<tr>
<td>3.0</td>
<td>5.0</td>
<td>7.0</td>
<td>1.0</td>
<td>1.0</td>
<td>3.0</td>
<td>0.0</td>
<td>2.0</td>
<td>6.0</td>
<td>1.0</td>
<td>7.0</td>
<td>9.0</td>
<td>5.0</td>
<td>1.0</td>
<td>4.0</td>
</tr>
<tr>
<td>2.0</td>
<td>1.0</td>
<td>7.0</td>
<td>6.0</td>
<td>1.0</td>
<td>5.0</td>
<td>2.0</td>
<td>0.0</td>
<td>9.0</td>
<td>4.0</td>
<td>2.0</td>
<td>1.0</td>
<td>1.0</td>
<td>7.0</td>
<td>8.0</td>
</tr>
<tr>
<td>1.0</td>
<td>3.0</td>
<td>1.0</td>
<td>5.0</td>
<td>2.0</td>
<td>4.0</td>
<td>6.0</td>
<td>9.0</td>
<td>0.0</td>
<td>3.0</td>
<td>3.0</td>
<td>5.0</td>
<td>1.0</td>
<td>6.0</td>
<td>4.0</td>
</tr>
<tr>
<td>5.0</td>
<td>4.0</td>
<td>6.0</td>
<td>4.0</td>
<td>1.0</td>
<td>7.0</td>
<td>1.0</td>
<td>4.0</td>
<td>3.0</td>
<td>0.0</td>
<td>9.0</td>
<td>1.0</td>
<td>8.0</td>
<td>5.0</td>
<td>2.0</td>
</tr>
<tr>
<td>7.0</td>
<td>6.0</td>
<td>5.0</td>
<td>2.0</td>
<td>3.0</td>
<td>8.0</td>
<td>7.0</td>
<td>2.0</td>
<td>3.0</td>
<td>9.0</td>
<td>0.0</td>
<td>2.0</td>
<td>1.0</td>
<td>8.0</td>
<td>1.0</td>
</tr>
<tr>
<td>1.0</td>
<td>6.0</td>
<td>9.0</td>
<td>1.0</td>
<td>6.0</td>
<td>3.0</td>
<td>9.0</td>
<td>1.0</td>
<td>5.0</td>
<td>1.0</td>
<td>2.0</td>
<td>0.0</td>
<td>5.0</td>
<td>4.0</td>
<td>3.0</td>
</tr>
<tr>
<td>2.0</td>
<td>6.0</td>
<td>1.0</td>
<td>2.0</td>
<td>8.0</td>
<td>1.0</td>
<td>5.0</td>
<td>1.0</td>
<td>1.0</td>
<td>8.0</td>
<td>1.0</td>
<td>5.0</td>
<td>0.0</td>
<td>9.0</td>
<td>6.0</td>
</tr>
<tr>
<td>9.0</td>
<td>1.0</td>
<td>3.0</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
<td>1.0</td>
<td>7.0</td>
<td>6.0</td>
<td>5.0</td>
<td>8.0</td>
<td>4.0</td>
<td>9.0</td>
<td>0.0</td>
<td>7.0</td>
</tr>
<tr>
<td>3.0</td>
<td>9.0</td>
<td>4.0</td>
<td>3.0</td>
<td>5.0</td>
<td>5.0</td>
<td>4.0</td>
<td>8.0</td>
<td>4.0</td>
<td>2.0</td>
<td>1.0</td>
<td>3.0</td>
<td>6.0</td>
<td>7.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
TSP GA in operation (3)

Improvement in generation 1: 25: 4 6 10 14 9 11 7 12 3 5 13 1 0 8 2
Improvement in generation 9: 22: 4 6 14 9 11 7 12 10 3 5 13 1 0 8 2
Improvement in generation 10: 20: 4 6 9 14 10 11 7 12 3 5 13 1 0 8 2
Improvement in generation 30: 18: 4 6 9 14 10 12 7 11 3 5 13 1 0 8 2
Improvement in generation 317: 17: 4 6 9 14 10 12 5 13 3 11 7 1 0 8 2

best after 1000 generations, 17: 4 6 9 14 10 12 5 13 3 11 7 1 0 8 2

verification: 4 (1.0) 6 (1.0) 9 (2.0) 14 (1.0) 10 (1.0) 12 (1.0) 5 (2.0) 13 (1.0) 3 (1.0) 11 (1.0) 7 (1.0) 1 (1.0) 0 (1.0) 8 (1.0) 2 (1.0) 4

Roulette-Wheel Optimization

- Rather than keep n copies of the same individual, record the individual once, along with its % of the population.

- Then during selection, choose individuals by spinning a “roulette wheel” biased with the given % toward the individual.
Like gradient descent, GA’s can also get stuck in local fitness extrema.

The space is different; for GA’s, a stuck point corresponds to a population from which crossover does not yield any better individuals.

Mutation is one hope for leaving such an extremum. Other possibilities are simulated annealing, random restarts.
Evolution Options

- Since we are simulating using a computer, not actually evolving species, there is no reason why Lamarckian, rather than Darwinian, evolution could not be used.

- Most results to date are Darwinian.

- Lamarckian could integrate other learning models to breed new species from individuals that have learned.

Lamarckian Leads


Genetic Programming

- Genetic programming is the GA idea applied to evolving programs (as opposed to just numbers).
- The prime mover of this field is John R. Koza.

Reference

John R. Koza

Genetic Programming: On the Programming of Computers by Means of Natural Selection

MIT Press, 1996
Genetic Programming Genomes = Syntax Trees

For animated tutorial, please see: http://www.genetic-programming.com/gpanimatedtutorial.html

Mutation of a Program
Crossover of Two Programs

Genetic Programming Demo: Symbolic Regression
(http://www.ifh.ee.ethz.ch/~gerber/approx/default.html)
The evolved program

GP Caution

- Typically fitness of genetically-evolved programs is established by working on a large number of test cases.
- We know this is not completely sound.
- A possible fruitful area is to evolve a proof of correctness along with the program itself. I know of no work in this area.
Engineering Applications: Evolving Hardware (Analog & Digital)

- The evolved program is a list of instructions for constructing the circuit, rather than the graph of the circuit itself.
- This approach has also been used to construct neural networks.
- The results for circuit design are competitive with, or superior to, human engineering.
- Numerous patents have been reinvented using GP.

Generating Analog Circuits

EETIMES

June 03, 1996, Issue: 904
Section: Technology

Genetic program auto-designs analog circuits
R. Colin Johnson

Palo Alto, Calif - Genetic algorithms (GA) can sometimes rescue software engineers stymied by problems that don't lend themselves to conventional programming techniques. But hardware engineers should be using GAs for designing analog circuits, too, claims Stanford professor John Koza.

"Genetic programming can design what you want if what you get electronic circuitry without any prior knowledge about electrical engineering," said Koza. He went on to contrast his approach with that of conventional artificial intelligence. "AI says that the power is in the knowledge, but I say that knowledge is the enemy."

"What we want to use is nature's method to evolve optimal solutions without the hindrance of preconceived knowledge."

An analog filter generated by genetic programming

Figure 7 The 100% compliant best circuit of generation 31 has the Cauer (elliptic) topology.

A Genetically-Evolved Amplifier

Figure 13 Best circuit from generation 109.
Koza (striped shirt) with 70-node Beowulf cluster

Koza’s 1000 node Beowulf used for genetic programming
Commercial Applications: Marketing Projections

http://www.statsoft.com/textbook/stcluan.html#h

Function Repertoire Menu
Other Opportunities

- Parallelism in Computation (local work of Beda & Margileth, and Tom Johnson, CS 152 steu)
- Parallelism optimizing transformations
- Music
- Robotics

Koza GP Video Clips