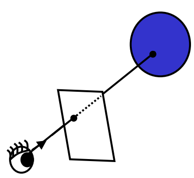
cs155 – z sweedyk

ray tracing

ray tracing

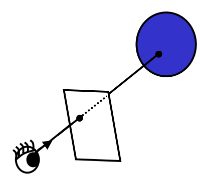
- simple ray casting
- recursive ray tracing
- modeling transforms
- cheap tricks
- optimizations

ray casting



- cast ray through pixel into scene
- find intersection point (if any) that is closest to eye
- compute luminance at intersection

ray casting

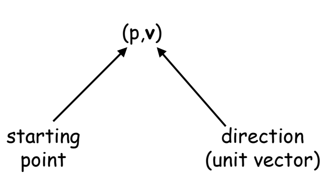


- **cast ray through pixel into scene**

What is the ray R_{ij} through pixel (i,j)?

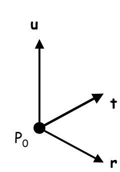
ray specification

a ray is a half-line defined by a point and a unit vector



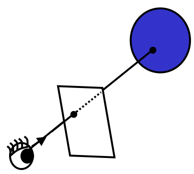
viewpoint specification: P_0, **u**, **t**

3d world coordinates



- P_0 is the viewpoint
- **u**, **t**, and **r**=**t**×**u** are orthogonal unit vectors in the up, toward, and right directions

ray casting



- **cast ray through pixel into scene**

What is the ray R_{ij} through pixel (i,j)?

$R_{ij} = (P_0, v_{ij})$

The next several slides are devoted to compute v_{ij}.

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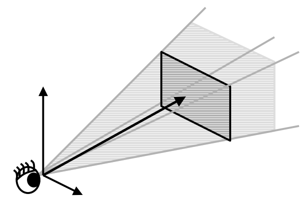
view volume



the world that is visible to the viewer is called the *view volume*. it is a pyramid that is aligned with the toward, up, and right vectors

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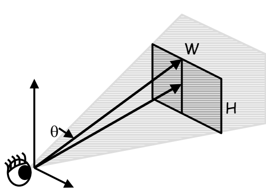
view volume specification: θ, ρ



the view volume is specified by the half-height angle θ and the aspect ratio ρ=W/H

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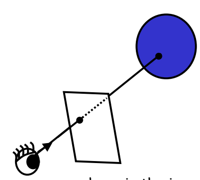
ray casting



- **cast ray through pixel into scene**

where is the image plane?

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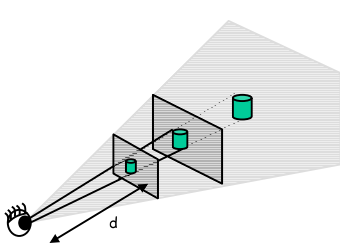
where is the image plane?



the choice of the image plane only affects the image size in world coordinates. for the moment assume it is d units from the viewpoint. we'll specify a convenient *d* in a few slides.

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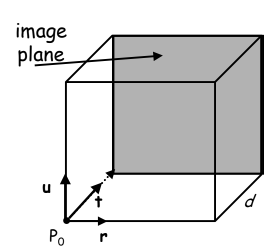
image plane



the image plane is orthogonal to **t** a distance *d* from P_o

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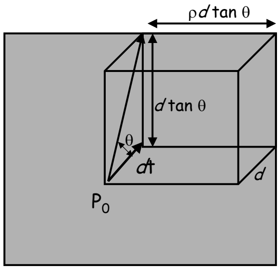
projected image in world coordinates



In world coordinates

- $P_0+d\mathbf{t}$ is the center of the image
- the image height is $H_d=2d\tan\theta$
- the image width is $W_d=\rho 2d\tan\theta$

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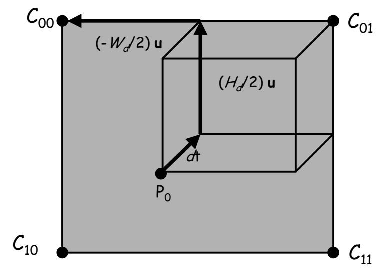
corner coordinates



$C_{00} = P_0 + d\mathbf{t} + (H_d/2)\,\mathbf{u} - (W_d/2)\,\mathbf{r}$

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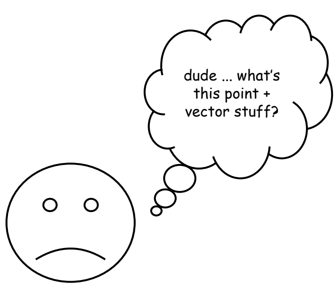

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point + vector

p + **v** is the point you get to by walking **v** from p.

$q=p+v = (p_x+v_x, p_y+v_y, p_z+v_z)$

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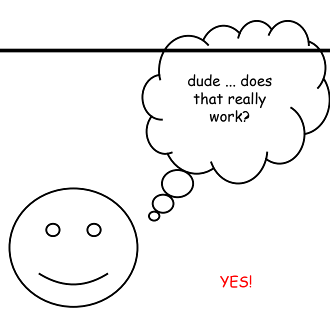

YES!

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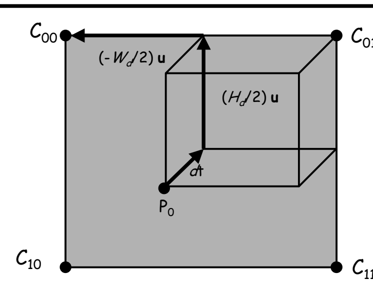
corner coordinates



$C_{00} = P_0 + d\mathbf{t} + (H_d/2)\,\mathbf{u} - (W_d/2)\,\mathbf{r}$

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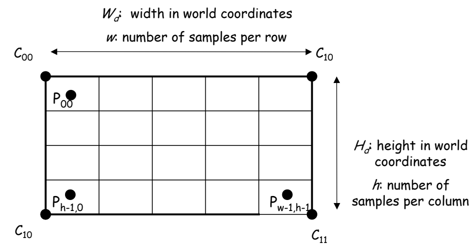
pixel coordinates



$$P_{ij} = C_{00} + (i+\tfrac{1}{2})(W_d/w)\ \mathbf{r} - (j+\tfrac{1}{2})(H_d/h)\ \mathbf{u}$$

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choosing d

$$P_{ij} = C_{00} + (i+\tfrac{1}{2})(W_d/w)\ \mathbf{r} - (j+\tfrac{1}{2})(H_d/h)\ \mathbf{u}$$

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specification

- Input specification
  - Viewpoint position P_0 and orientation **t**, **u**
  - Image width *w* and height *h* in pixels
  - Half-height angle θ
- Compute
  - Right vector $\mathbf{r} = \mathbf{t} \times \mathbf{u}$
  - Aspect ratio $\rho = w/h$
  - $H_d = h$, $W_d = w$
  - $d = h/\tan\theta$

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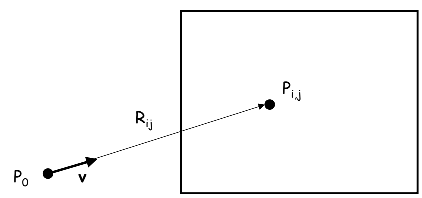
casting rays



$R_{ij} = (P_0, \mathbf{v}_{ij})$ where $\mathbf{v}_{ij} = (P_{i,j} - P_0)/||P_{i,j} - P_0||$

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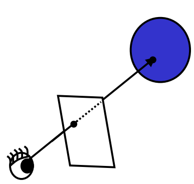
ray casting



- cast ray through pixel into scene
- **find intersection point (if any) that is closest to eye**
- compute luminance at intersection

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intersection

- sphere
- triangle
- box
- cylinder
- cone
- torus

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intersection

- **sphere**
- triangle
- box
- cylinder
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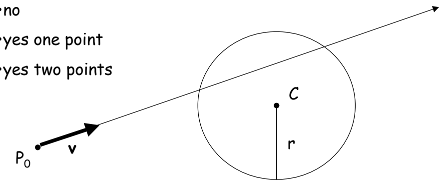
- torus

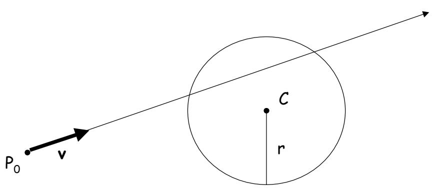
sphere intersection

- Is there a point that lies on the sphere and on the ray (P_0,**v**)?
- Possible answers
  - no
  - yes one point
  - yes two points



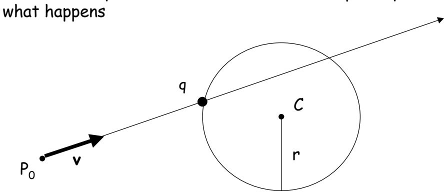
sphere intersection

- Is there a point that lies on the sphere and on the ray (P_0,**v**)?
- If so, choose the one closest to P_0.



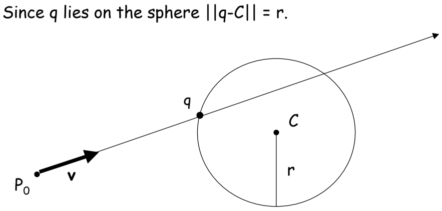
sphere intersection

- Is there a point that lies on the sphere and on the ray (P_0,**v**)?
- To answer this question we'll assume there is a point q and see what happens



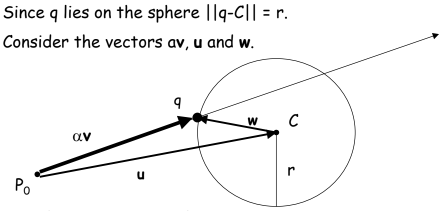
sphere intersection

- Since q lies on R, there is some $\alpha \geq 0$ such that $q = P_0 + \alpha \mathbf{v}$.
- Since q lies on the sphere $||q-C|| = r$.



sphere intersection

- Since q lies on R, there is some $\alpha \geq 0$ such that $q = P_0 + \alpha \mathbf{v}$.
- Since q lies on the sphere $||q-C|| = r$.
- Consider the vectors α**v**, **u** and **w**.



Then $\mathbf{w} = \alpha\mathbf{v} - \mathbf{u}$ and

$$r^2 = ||w||^2 = w \bullet w = (\alpha\mathbf{v} - \mathbf{u}) \bullet (\alpha\mathbf{v} - \mathbf{u})$$
$$= \alpha^2 ||\mathbf{v}||^2 - 2\alpha(\mathbf{u} \bullet \mathbf{v}) + ||\mathbf{u}||^2$$

sphere intersection

Does the ray intersect the sphere?

⇕

Does the quadratic (in α)

$$||\mathbf{v}||^2 \alpha^2 - 2 (\mathbf{u} \bullet \mathbf{v}) \alpha + ||\mathbf{u}||^2 - r^2$$

have any real, non-negative roots?

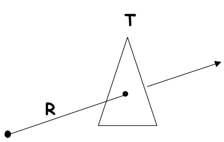
•No: no intersection

•Yes: intersection point is $q=P_0+r\mathbf{v}$ where r is the smallest non-negative root.

intersection

- sphere
- **triangle**
- box
- cylinder
- cone
- torus

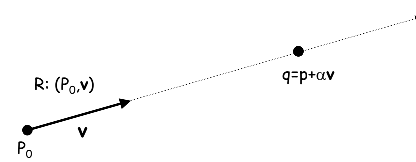
triangle intersection



- do R and T intersect?
- if so, where?

ray: parametric form

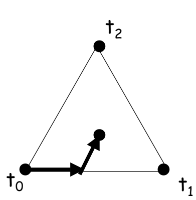
a point q lies on R=(p,**v**) iff $q=p+\alpha\mathbf{v}$ for some $\alpha \geq 0$



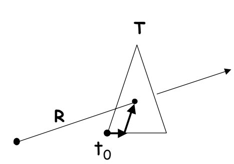
triangle: parametric form

a point q lies on the triangle T iff $q=t_0+\beta\mathbf{u}+\gamma\mathbf{w}$ where

$\mathbf{u} = t_1-t_0$, $\mathbf{w} = t_2 - t_0$, and $\beta \geq 0$, $\gamma \geq 0$, $\beta+\gamma \leq 1$



triangle intersection



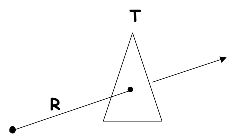
Find α, β, γ such that

$P_0+\alpha\mathbf{v} = t_0+\beta\mathbf{u}+\gamma\mathbf{w}$

$\alpha, \beta, \gamma \geq 0$

$\beta+\gamma \leq 1$

triangle intersection – take 1



Find α, β, γ such that

$P_0+\alpha\mathbf{v} = t_0+\beta\mathbf{u}+\gamma\mathbf{w}$

$\alpha, \beta, \gamma \geq 0$

$\beta+\gamma \leq 1$

System of linear equations

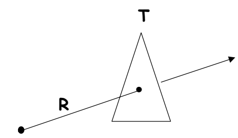
$P_{0x}+\alpha v_x = t_{0x}+\beta u_x+\gamma w_x$

$P_{0y}+\alpha v_y = t_{0y}+\beta u_y+\gamma w_y$

$P_{0z}+\alpha v_z = t_{0z}+\beta u_z+\gamma w_z$

Invert the matrix. Yuck. Is there another way?

triangle intersection – take 2



1. Find the intersection point (if any) of R and the plane containing T.
2. Find the parameterization of the intersection point in terms of β, γ.

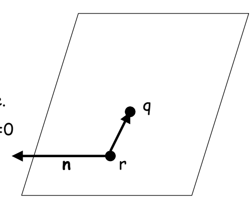
plane

Let **n** be a normal to the plane.

Let r be a point on the plane.

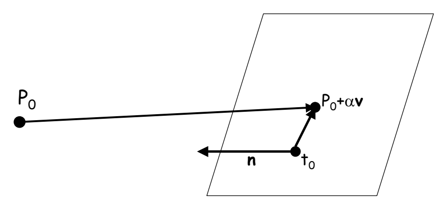
Let q be an arbitrary point in space.

Then q lies on the plane iff $\mathbf{n}\bullet(q-r)=0$



plane intersection

Is there a point $P_0+\alpha\mathbf{v}$, $a\geq 0$, such that $\mathbf{n}\bullet(P_0+\alpha\mathbf{v} - t_0) = 0$?



plane intersection

Is there a point $P_0+\alpha\mathbf{v}$, $a\geq 0$, such that $\mathbf{n}\bullet(P_0+\alpha\mathbf{v} - t_0) = 0$?

Assume there is and solve for α.

Let **r** be the vector from P_0 to t_0.

Then $0 = \mathbf{n}\bullet(P_0+\alpha\mathbf{v} - t_0) = \mathbf{n}\bullet(\alpha\mathbf{v} - \mathbf{r}) = \alpha(\mathbf{n}\bullet\mathbf{v}) - \mathbf{n}\bullet\mathbf{r}$.

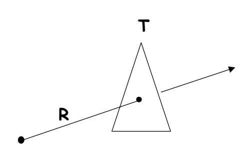
If $\mathbf{n}\bullet\mathbf{v}=0$ then R is either parallel to plane or it lies in the plane.

If $\mathbf{n}\bullet\mathbf{v}\neq 0$ then $\alpha = (\mathbf{n}\bullet\mathbf{r})/(\mathbf{n}\bullet\mathbf{v})$.

If $\alpha\geq 0$ then R intersects the plane at point $P_0+\alpha\mathbf{v}$.

If $\alpha<0$ then R does not intersect plane.

triangle intersection – take 2



1. Find the intersection point (if any) of R and the plane containing T.
2. Find the parameterization of the intersection point in terms of β, γ.

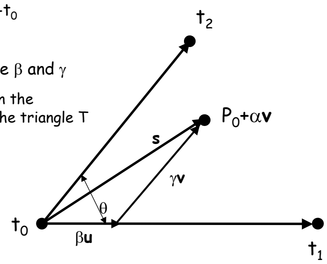
triangle intersection

recall $\mathbf{u}=t_1-t_0$ and $\mathbf{w}=t_2-t_0$

let $\mathbf{s} = P_0+av - t_0$.

then $\mathbf{s}= \beta\mathbf{u} +\gamma\mathbf{w}$ for some β and γ

if $\beta, \gamma \geq 0$ and $\beta+\gamma\leq 1$ then the intersection point lies in the triangle T



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triangle intersection

To find β and γ solve system of equations:

$\beta(\mathbf{u} \bullet \mathbf{w}) + \gamma(\mathbf{w} \bullet \mathbf{w}) = \mathbf{s} \bullet \mathbf{w}$

$\beta(\mathbf{u} \bullet \mathbf{u}) + \gamma(\mathbf{w} \bullet \mathbf{u}) = \mathbf{s} \bullet \mathbf{u}$

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intersection

- triangle
- sphere
- box
- cylinder
- cone
- torus

Etc.

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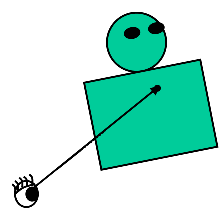
ray tracing



- cast ray into scene
- **find intersection point (if any) that is closest to eye**
- compute luminance at intersection

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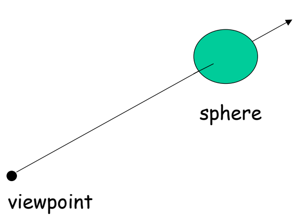
find intersection point



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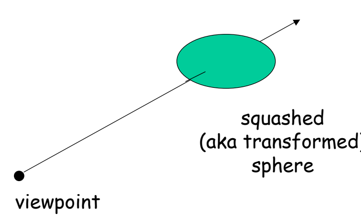
find intersection point



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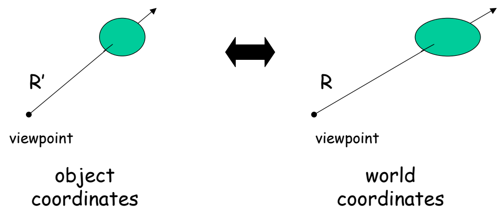
find intersection point



does this make sense?

- is there an inverse transform?
- how do we apply a transform to a ray?
- is a ray in world coordinates a ray in object coordinates?

Conceptually: scale

What operation inverts a scale by s in the x-direction?

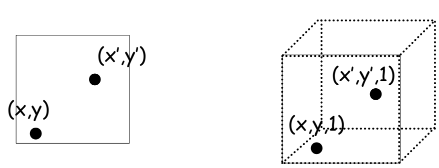
For $s \neq 0$, scale by 1/s in the x-direction.

Any problem?

We are not alone!

we are not alone…

the parallel universe view of homogenous coordinates



we live in this universe

it's not the only one, but it is the only one we can experience!

scale

$$\begin{pmatrix} s^{-1} & 0 & 0 & 0 \\ 0 & t^{-1} & 0 & 0 \\ 0 & 0 & u^{-1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = ?$$

Conceptually: rotate

What operation inverts a rotate by θ about the x-axis?

Rotate by $-\theta$ about the x-axis.

rotate about z axis

$$\begin{pmatrix} \cos -\phi & -\sin -\phi & 0 & 0 \\ \sin -\phi & \cos -\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \phi & -\sin \phi & 0 & 0 \\ \sin \phi & \cos \phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =?$$

remember $\cos(-\phi) = \cos(\phi)$ and $\sin(-\phi) = -\sin(\phi)$

rotate about z axis

$$\begin{pmatrix} \cos \phi & \sin \phi & 0 & 0 \\ -\sin \phi & \cos \phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \phi & -\sin \phi & 0 & 0 \\ \sin \phi & \cos \phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =?$$

remember $\cos^2 \phi + \sin^2 \phi = 1$

Conceptually: translate

What operation inverts a translate by dx in the x-direction?

Translate by -dx in the x-direction.

translate

$$\begin{pmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 1 & z_0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =?$$

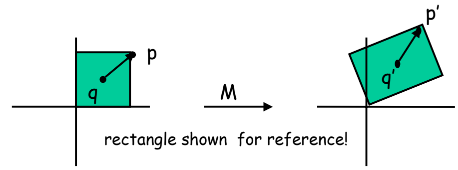
does this make sense?

- is there an inverse transform?
- how do we apply a transform to a ray?
- is a ray in world coordinates a ray in object coordinates?

transform

- Points Done!!
- Vectors
- Rays

tranforms: vector



v=p-q and T(**v**)=T(p-q)=T(p)-T(q) = Mp-Mq

Warnings:

- because of translation we can't ignore q=(0,0,0)
- re-unitize unit vectors

transforms

- Points Done!!
- Vectors Done!!
- **Rays**

transforms: ray

- Points
- Vectors
- **Rays: r=(p,v)**

Transform point
and
transform/unitize
vector!

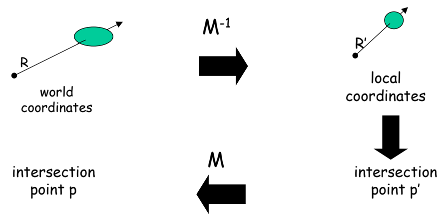
does this make sense?

- is there an inverse transform?
- how do we apply a transform to a ray?
- is a ray in world coordinates a ray in object coordinates?

Linear transforms

Linear transforms preserve lines!

find intersection point



M and M^{-1}

single transform

M

- scale by s
- rotate by θ
- translate by Δ

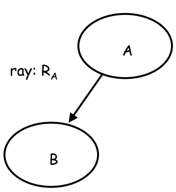
M^{-1}

- scale by 1/s
- rotate by $-\theta$
- translate by $-\Delta$

composite transform

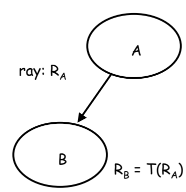
$(M_1M_2 \ldots M_k)^{-1}$ $\quad$ $M_k^{-1} \ldots M_2^{-1}M_1^{-1}$

scene graph traversal



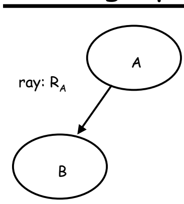
A sends the ray (represented relative to A's coordinate system) to B.

scene graph traversal



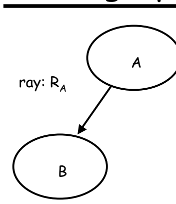
B converts the ray into its own coordinate system

scene graph traversal



B computes the intersections of R_B with its objects

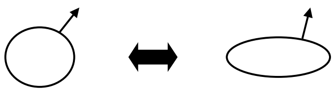
scene graph traversal



- B sends R_B to its descendents
- Each returns intersection information (represented in B's coordinate system)
- B chooses closest to viewer
- B converts intersection info to A's coordinate system and returns it

surface normal

is the normal to a
transformed surface the
transformed normal?



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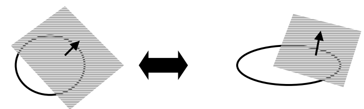
surface normal

is the tangent plane to a
transformed surface the
transformed tangent plane?



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The right way …

N is normal to the tangent plane iff for any points p and q on the tangent plane $N^T \bullet (p-q) = 0$.

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The right way …

N is normal to the tangent plane iff for any points p and q on the tangent plane $N^T \bullet (p-q) = 0$.

Assume N is normal to the tangent plane and QN is normal to the tangent plane transformed by M.

Q must satisfy the following for any points p and q on the tangent plane:

$N^T(p-q)=0$ iff $(QN)^T(M(p-q))=0$

⬍

$N^T(p-q)=0$ iff $N^T(Q^T M)(p-q)=0$

Thus $Q=(M^{-1})^T$

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