Natural Deduction Worked Problems

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HR 1.4.1 a

- Derive ⊢ (p → q) → p

1.	(p → q)	Assumption
2.	p	→e 1
3.	(p → q) → p	→ i 1-2

HR 1.4.1 c

- Derive $p \vdash (p \rightarrow q) \rightarrow q$

1.	p	Premise
2.	$(p \rightarrow q)$	Assumption
3.	q	$\rightarrow$ e 1, 2
4.	$(p \rightarrow q) \rightarrow q$	$\rightarrow$ i 2-3

HR 1.4.2 a

- Derive $(p \rightarrow r) \wedge (q \rightarrow r) \vdash (p \vee q) \rightarrow r$

1.	$(p \rightarrow r) \wedge (q \rightarrow r)$	Premise
2.	$(p \rightarrow r)$	$\wedge e_1$ 1
3.	$(p \vee q)$	Assumption
4.	p	$\wedge e_1$ 3
5.	r	$\rightarrow e$ 2, 5
6.	$(p \vee q) \rightarrow r$	$\rightarrow i$ 3-5

HR 1.4.2 b

- Derive $(q \rightarrow r) \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$

1.	$(q \rightarrow r)$	Premise
2.	$(p \rightarrow q)$	Assumption
3.	p	Assumption
4.	q	$\rightarrow$ e 3, 2
5.	r	$\rightarrow$ e 4, 1
6.	$p \rightarrow r$	$\rightarrow$ i 3-5
7.	$(p \rightarrow q) \rightarrow (p \rightarrow r)$	$\rightarrow$ i 2-6

HR 1.4.2 c

- Derive p → (q → r), (p → q) ⊢ (p → r)

1.	p → (q → r)	Premise
2.	(p → q)	Premise
3.	p	Assumption
4.	q	→ e 3, 2
5.	q → r	→ e 3, 1
6.	r	→ e 4, 5
7.	(p → r)	→ i 3-6

HR 1.5.2 a

- Derive ¬p → p ⊢ p

1.	¬p → p	Premise
2.	p ∨ ¬p	LEM
3.	¬p	Assumption
4.	p	→ e 3, 1
5.	p	Assumption
6.	p	∨ e 2 , 5, 3-4

HR 1.5.2 b

- Derive □p |□ p □ q

1.	□p	Premise
2.	p	Assumption
3.	□	□e 2, 1
4.	q	□e 4
5.	p □ q	□ i 2-4

HR 1.5.2 c

- Derive p ∨ q, ¬q ⊢ p

1.	p ∨ q	Premise
2.	¬q	Premise
3.	q	Assumption
4.	⊥	¬e 3, 2
5.	p	⊥e 4
6.	p	Assumption
7.	p	∨e 1, 6-6, 3-5

HR 1.5.2 d

- Derive |□ □p □ (p □ (p □ q))

1.	□p	Assumption
2.	p	Assumption
3.	□	□e 3, 2
4.	(p □ q)	□e 4
5.	(p □ (p □ q))	□ i 2-4
6.	□p □ (p □ (p □ q))	□ i 1-5

HR 1.5.2 e

- Derive $\vdash (p \to q) \vee (q \to p)$

1.	$q \vee \neg q$	LEM
2.	$\quad q$	Assumption
3.	$\quad p$	Assumption
4.	$\quad q$	2
5.	$\quad (p \to q)$	$\to$ i 3-4
6.	$\quad (p \to q) \vee (q \to p)$	$\vee_1$ i 5
7.	$\quad \neg q$	Assumption
8.	$\quad\quad q$	Assumption
9.	$\quad\quad \bot$	$\neg$e 3, 2
10.	$\quad\quad (q \to p)$	$\bot$e 9
11.	$\quad (p \to q) \vee (q \to p)$	$\vee_2$ i 10
12.	$(p \to q) \vee (q \to p)$	e 1, 2-6, 7-11