Algorithms, Section 2
Spring 2003
Homework 12a
Due Thursday, April 24

1. [**10 Points**] **Professor Lai and NP!**
   (a) Professor I. Lai from P.I.T. claims that if P_1 and P_2 are any two problems in NP such that $P_1 \leq_p P_2$, then $P_2 \leq_p P_1$. Prove that this would imply that $P = NP$.
   (b) Professor Lai is studying an interesting problem called the Polygon Intersection Testing problem (PIT). He claims to have proved that PIT is NP-complete by showing that PIT is in NP and that PIT $\leq_p$ VC (VC is the Vertex Cover problem). Is this a valid approach for proving that PIT is NP-complete? Explain.

2. [**20 Points**] **HITTING SET!** First, the gratuitous story: The Computer Science Department at The Western Institute of Technology (T.W.I.T.) has 5 faculty members, Dr. I. "Theo" Rize, Dr. I. Dutu, Dr. Sam Heer, Dr. Mia Thue, and Dr. Juan More. Each professor serves on a number of committees. The Polynomial Time Algorithms Committee consists of Professors Rize, Dutu, and More. The NP-Completeness Committee consists of Professors Dutu, Heer, and Thue. Finally, the Other Stuff Committee consists of Professors Thue and More. The Dean would like to convene a meeting in which the number of computer scientists invited is minimized (she can't stand their bad CS jokes!) but each committee is represented by at least one of its members. For example, inviting Professors Thue and More would be a solution in this case.

   The Dean has formulated this problem as the following general problem (known to computer scientists as the HITTING SET Problem): We are given a set S (the set of professors) and a collection, C, of subsets of S (the set of committees). A hitting set for C is a subset $S' \subseteq S$ such that S' contains at least one element from each subset in C. The HITTING SET optimization problem is to find the smallest hitting set.

   (a) Formulate the HITTING SET decision problem corresponding to the optimization problem described above.
   (b) In this part of the problem, you will prove that the HITTING SET decision problem is NP-complete.
      i. The first part of proving NP-completeness is to show that HITTING SET is in the class NP. Briefly explain why this is true.
      ii. Next, we need to give a reduction from some known NP-complete problem to the HITTING SET problem. Describe a reduction from the Vertex Cover Problem.
      iii. Next, explain why this reduction can be performed in polynomial time.
      iv. Finally, prove that your reduction is correct. That is, you need to show that the answer to an instance of VERTEX COVER is true if and only if (that requires two separate proofs!) the answer to the constructed HITTING SET problem is true.