Algorithms
Computer Science 140 & Mathematics 168
Fall 2003
Instructor: B Thom
Homework 1a
Due Thursday, Sept. 4

- The reading material for this week is Chapters 1, 2, and 3 of CLRS.
- Please recall that homeworks are due at the *beginning of the next class.*
- *This homework set is intended to review a few mathematical fundamentals that we'll be using extensively.*
- *Bonus problems are optional and* should be turned in on a separate sheet.
- An important part of being a computer scientist or mathematician is the ability to express solutions clearly and thoroughly. Therefore, you are expected to explain each step of your solution and to write your solutions neatly. The graders will deduct points for missing steps or solutions which they cannot easily read.
- The homeworks sets are also posted on the course web page, both in pdf format as well as in LaTeX. You are encouraged (but by no means required) to submit your homeworks in LaTeX.

1. **[15 points] Properties of Logs.**
   Let's begin by proving that $\log_b xy = \log_b x + \log_b y$. The proof goes as follows: Let $k = \log_b xy$, $\ell = \log_b x$, and $m = \log_b y$. Then, $b^k = xy$, $b^\ell = x$, and $b^m = y$, by the definition of the logarithm. Thus, $b^k = b^\ell b^m = b^{\ell+m}$ by properties of exponents. Thus, $k = \ell + m$, which is what we had set out to prove.

   Now give proofs for each of the following properties of logarithms. Write your proofs out carefully. You should assume that a, b, c, n are positive *real numbers* (not necessarily integers).

   (a) $\log_b a^n = n \log_b a$.
   (b) $\log_b a = \frac{\log_c a}{\log_c b}$.
   (c) $a^{\log_b n} = n^{\log_b a}$.

2. **[10 points] The Geometric Series.**

   (a) Use induction on n to show that for all integers $n \geq 0$

   $$1 + a + a^2 + a^3 + \ldots + a^n = \frac{a^{n+1} - 1}{a - 1}$$

   where a is some arbitrary real number other than 1.

(b) Explain where your induction proof relied on the fact that $a \neq 1$.

3. **[20 points] OPTIONAL BONUS PROBLEM! The Arithmetic and Geometric Means.** Let $x_1, \ldots, x_n$ be positive real numbers. The *arithmetic mean* of these numbers is defined to be $\frac{x_1+x_2+\ldots+x_n}{n}$ and the *geometric mean* is defined to be $(x_1 x_2 \cdots x_n)^{1/n}$. In this problem we show that the arithmetic mean of n numbers is at least as large as the geometric mean of those numbers.

   (a) Use induction to show that if $x_1 x_2 \cdots x_n = 1$ then $x_1 + x_2 + \ldots + x_n \geq n$. Observe that this is a special case of the statement we are trying to prove.

   (b) Use this fact to show that the arithmetic mean is at least as large as the geometric mean. (No induction required here; just a little algebra.)