Algorithms, Section 2
Spring 2003
Homework 8a
Due Thursday, March 27

This following two problems are challenging but important; please *start thinking about them right away.* By all means, come and talk to Belinda if you have any questions about the graph algorithms we've discussed so far in class (or if you need candy, conversation, a skate-boarding lesson, etc.). In addition to regular office hours, the usual Wednesday night time is available; I'll be in my office around 9:30 and leave before 10pm if no one shows up.

1. [**20 Points**] **Diameters.** In this problem we will consider connected undirected graphs with no cycles. There are no weights on the edges. The graphs are simply represented with adjacency lists. The distance between two vertices in such a graph is just the minimum number of edges on a path between the vertices. The *diameter* of the graph is the maximum distance between all pairs of vertices. Make sure that you understand this definition well before proceeding! Now, describe an algorithm for computing the diameter of a connected undirected graph with no cycles. Carefully analyze and explain the running time of your algorithm. *For full credit, your algorithm should be as fast as possible.*

2. [**25 points**] **An Algorithmic Get-Rich-Quick Scheme!** Your job at Millisoft was fun for awhile, but your reputation has gotten out and you've been hired back by the brokerage firm of Weil, Proffet, and Howe at a whopping salary. (We won't be specific about the actual amount, but let's just say that your salary is most easily expressed using scientific notation!)

   Weil, Proffet, and Howe has just entered the **arbitrage** business. Arbitrage is a money-making scheme involving anomalies in international currency exchange rates. For example, imagine that 1 U.S. dollar buys 0.8 Zambian kwachas, 1 Zambian kwacha buys 10 Mongolian tughriks, and 1 Mongolian tughrik buys 0.15 U.S. dollars. Then, by converting currencies, a trader can start with 1 U.S. dollar and buy $0.8 \times 10 \times 0.15 = 1.2$ U.S. dollars. By capitalizing on such anomalies quickly (before they're detected and corrected by the markets), huge amounts of money can be made.

   We'll assume that we're given n currencies and the exchange rate between every pair of currencies. That is, we are given currencies $c_1, \ldots, c_n$ and an $n \times n$ table R such that one unit of currency c_i buys $R[i, j]$ units of currency c_j.

   (a) We'll begin with the following problem. Given the list of currencies and exchange rates and a particular currency c_i, determine the maximum amount of each currency that you can obtain, beginning with 1 unit of currency c_i. *For this part of the problem assume that there exist no cycles that allow you to get arbitrarily rich via arbitrage.* Describe *two* different algorithms for doing this - one based on a modification of Floyd-Warshall and one based on a modification of Bellman-Ford. (Your algorithms can be described in English. Pseudo-code is not necessary.)

Explain briefly why each algorithm works. Finally, explain the running times of each of the two algorithms.

(b) Now assume that the exchange rates are such that it may be possible to get rich via arbitrage. That is, there may exist a cycle of currency exchanges that allows you to make more of your starting currency than you had initially. Describe how you could modify each of your algorithms to determine if it's possible to profit from arbitrage with the given currencies and exchange rates. Prove that each of your modified algorithm works.

(c) Finally, describe and analyze the running time of an algorithm for finding the actual sequence of currencies to be exchanged that makes a profit using arbitrage, if such a cycle exists.