Algorithms Section 2
Spring 2003
Homework 7
Due Tuesday, April 1 (no fooling!)

1. **[15 Points] Proof of Dijkstra's Algorithm.** Write the proof of correctness of Dijkstra's Algorithm (that we saw in class) in your own words. For full credit, your proof should be clear, precise, and complete. Note that the proof we did in class is not the same that is in the book. (The technique shown in class was much simpler; use that technique here.)

2. **[25 Points] Hurts' On-board Navigation System Revisited.** Hurts Car Rental has designed a new generation of alternative fuel vehicles. The new vehicles use a special fuel comprising a finely minced mixture of the "Introduction to Algorithms" textbook, chocolate fudge Pop Tarts, Spam, Spam, Spam, and Mountain Dew. Due to this rather unusual fuel requirement, there are only certain cities in the country where the vehicles can be refueled. Thus, to get from the start city to the destination city, the driver must plan a route that ensures that the car can be refueled along the way.

   The on-board computer on such a vehicle contains a weighted directed graph in which the vertices represent cities, the directed edges represent one-way roads, and the weights on the edges represent the lengths of the roads. Moreover, the computer knows which select cities have filling stations. To use the navigation system, the user will enter the starting city, the destination city, and the range of the vehicle on a full tank. (You may assume that the starting city and the destination cities, being Hurts rental cities, both have filling stations - and that the car starts with a full tank of fuel.) The computer will respond with the *shortest* route from the source to the destination that ensures that the vehicle doesn't run out of fuel or determines that no such route exists.

   You should assume that there is some constant C such that every vertex has in-degree and out-degree (number of entering edges and number of exiting edges, respectively) at most C.

   (a) **Describe** and **analyze the running time** of an efficient algorithm for determining such a path. For full credit, your algorithm should be as fast as possible.

   (b) *Prove* that your algorithm finds an optimal solution. Be careful and precise.

   (Note: Your algorithm may be built up of several algorithms which we have seen in class. You may apply any algorithms that we have seen in class.)

3. **[10 Points]** CLRS, Exercise 25.3-3 (page 640). Give a mathematical proof of your answer!

4. **[20 Points]** CLRS, Exercise 25.3-6 (page 640). The first part of this problem requires a single counter-example. The second part requires a careful proof. Clarification: the book uses `line 1` to refer to the entire action of creating the dummy vertex and adding in its zero weight edges. What Prof. Michener is proposing is to eliminate the dummy

vertex altogether (this means no additional edges are added), instead running Dijkstra on the original graph V times, once with each vertex being set to s.