cs155 – z sweedyk

new and used
mathematics

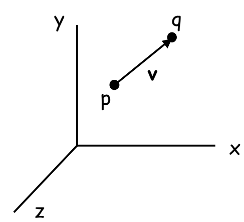
background: spaces

- linear space
  - scalar
  - vectors
- affine space
  - scalar
  - vector
  - points

scalars: real numbers

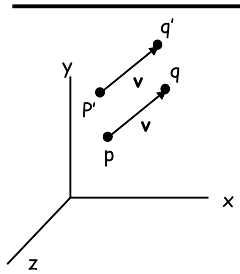
3.8
2.7
4.1
-1000.2
5

vector: distance & direction in (3d) space



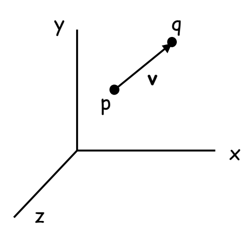
v: the way you get from p to q

vector: magnitude & direction in (3d) space



A vector does not have a position in space !

vector: distance & direction in (3d) space

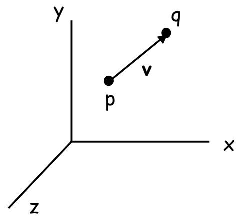


v: the way you get from p to q

$\mathbf{v} = \langle q_x - p_x, q_y - p_y, q_z - p_z \rangle$

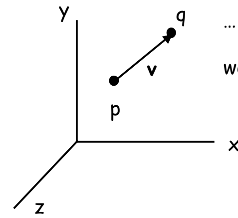
vector: notation

abusive but useful notation **v**=q-p



vector: distance & direction in (3d) space

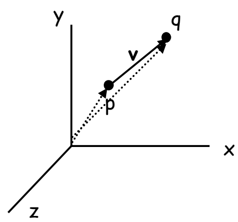
abusive but useful notation **v**=q-p



… abusive because addition of points is not defined and, even if it was, we'd expect the result to be a point, not a vector

vector: distance & direction in (3d) space

abusive but useful notation **v**=q-p

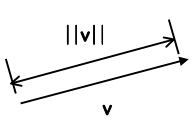


this notation makes perfect sense if we think of q and p as vectors

vector operations

- vector norm
- scalar multiplication
- vector addition
- dot product
- cross product (for 3d)
- linear transforms

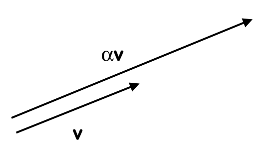
magnitude, length, euclidean norm



- $\mathbf{v}=\langle v_x, v_y, v_z \rangle$
- $||\mathbf{v}||=(v_x^2+ v_y^2+ v_z^2)^{\frac{1}{2}}$

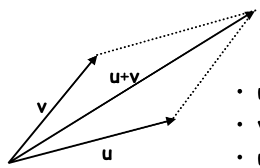
v is a unit vector if $||\mathbf{v}||=1$

scalar multiplication



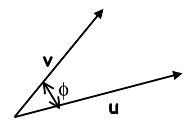
- $\mathbf{v}=\langle v_x, v_y, v_z \rangle$
- $\alpha v = \langle \alpha v_x, \alpha v_y, \alpha v_z \rangle$

vector addition



- **u**=<u_x,u_y,u_z>
- **v**=<v_x,v_y,v_z>
- **u+v**=< u_x+ v_x, u_y+ v_y, u_z +v_z>

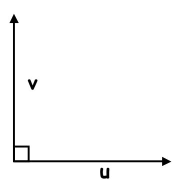
dot (inner) product



- **u**=<u_x,u_y,u_z>
- **v**=<v_x,v_y,v_z>
- **u•v**= $u_xv_x+u_yv_y+u_zv_z$
  = ||**u**||*||**v**|| cos ϕ

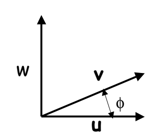
note: **v•v**=||**v**||^2

orthogonal vectors



- non-zero vectors **u** and **v** are orthogonal if **u•v** = 0

cross product (3d)

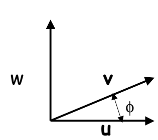


u X **v** = **w**

magnitude: ||**w**|| = ||**u**||*||**v**|| sin ϕ

direction: **w** is orthogonal to both u and v in direction defined by right hand rule

cross product (3d)



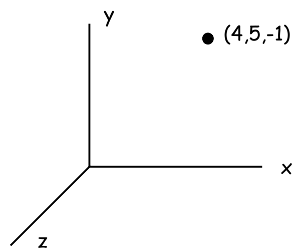
- **u**=<u_x,u_y,u_z>
- **v**=<v_x,v_y,v_z>

$$w = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$

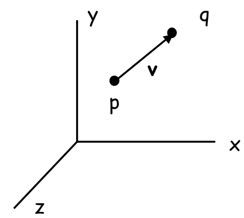
background: spaces

- linear space
  - scalar
  - vectors
- affine space
  - scalar
  - vector
  - points

point: position in (3d) space



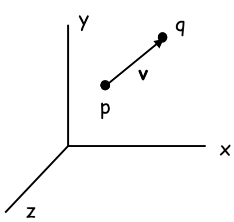
vector operating on a point



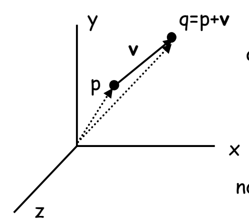
- $p=(p_x,p_y,p_z)$
- $\mathbf{v}=\langle v_x,v_y,v_z \rangle$
- $q=(p_x+v_x, p_y+v_y, p_z+v_z)$

vector operating on a point

abusive but useful notation
$q=p+\mathbf{v}$



vector operating on a point



abusive but useful notation
$q=p+\mathbf{v}$

notation makes sense if p and q are vectors