

cs155 - z sweedyk

ray tracing

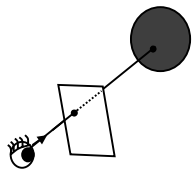
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ray tracing

- simple ray casting
- recursive ray tracing
- modeling transforms
- cheap tricks
- optimizations

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ray casting

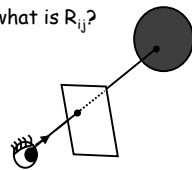


- cast ray through pixel into scene
- find intersection point (if any) that is closest to eye
- compute luminance at intersection

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ray casting

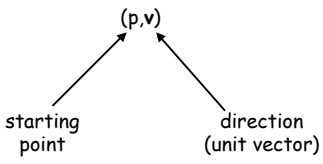
what is R_{ij} ?



- **cast ray through pixel into scene**
- find intersection point (if any) that is closest to eye
- compute luminance at intersection

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ray specification




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viewpoint orientation

3d world coordinates

- P_o is the viewpoint
- u , t , and r are unit vectors (typically orthogonal) in the up, toward, and right directions



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ray casting

$R_{ij} = (P_0, v_{ij})$
what is v_{ij} ?

- cast ray through pixel into scene
- find intersection point (if any) that is closest to eye
- compute luminance at intersection

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image plane

the image plane is orthogonal to \mathbf{t} a distance d from P_0 .

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where is the view plane?

the choice of d only affects the image size in world coordinates. we'll specify a convenient d in a few slides.

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projected image in world coordinates

- suppose the view volume is specified by a half-height θ angle and aspect ratio p
- $P_0 + d\mathbf{t}$ is the center of the image
- in world coordinates
 - image height is $H_f = 2d \tan \theta$
 - image width is $W_f = p 2d \tan \theta$

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corner coordinates

$C_{00} = P_0 + d\mathbf{t} + (H_f/2)\mathbf{u} - (W_f/2)\mathbf{r}$

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pixel coordinates

$P_{ij} = C_{00} + (i + \frac{1}{2})(W_d/w)\mathbf{r} - (j + \frac{1}{2})(H_d/h)\mathbf{u}$

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choosing d

$$P_{ij} = C_{00} + (i + \frac{1}{2})(W_d/w) \mathbf{r} - (j + \frac{1}{2})(H_d/h) \mathbf{u}$$

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specification

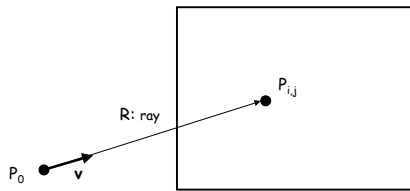
- Input specification
 - Viewpoint position P_0 and orientation \mathbf{t}, \mathbf{u}
 - Image width w and height h in pixels
 - Half-height angle θ
- Compute
 - Right vector $\mathbf{r} = \mathbf{t} \times \mathbf{u}$
 - Aspect ratio $\rho = w/h$
 - $H_d = h, W_d = w$
 - $d = h/\tan \theta$

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casting rays



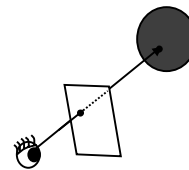
$$R = (P_0, \mathbf{v}) \text{ where } \mathbf{v} = (P_{ij} - P_0) / \|P_{ij} - P_0\|$$

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ray casting



- cast ray through pixel into scene
- **find intersection point (if any) that is closest to eye**
- compute luminance at intersection

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intersection

- sphere
- triangle
- box
- cylinder
- cone
- torus

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intersection

- **sphere**
- triangle
- box
- cylinder
- cone
- torus

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sphere intersection

- Is there a point that lies on the sphere and on the ray (P_0, v) ?
- Possible answers
 - no
 - yes one point
 - yes two points

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sphere intersection

- Is there a point that lies on the sphere and on the ray (P_0, v) ?
- If so, choose the one closest to P_0 .

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sphere intersection

- Is there a point that lies on the sphere and on the ray (P_0, v) ?
- To answer this question we'll assume there is a point q and see what happens

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ray: parametric form

q lies on $R=(p,v)$ iff $q=p+\alpha v$ for some $\alpha \geq 0$

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sphere intersection

- Is there a point $q = P_0 + \alpha v$, $\alpha \geq 0$, such that $\|q - C\| = r$?
- Let's assume there is one and solve for α .

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sphere intersection

- Is there a point that lies on the sphere and on the ray (P_0, v) ?
- To answer this question we'll assume there is a point q and see what happens.
- Consider the vectors αv , u and w .

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sphere intersection

Then $w = \alpha v - u$ and

$$r^2 = \|w\|^2 = w \cdot w = (\alpha v - u) \cdot (\alpha v - u) \\ = \alpha^2 \|v\|^2 - 2\alpha(u \cdot v) + \|u\|^2$$

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sphere intersection

• Is there a point $q = P_0 + \alpha v$, $\alpha \geq 0$, such that $\|q - C\| = r$?

• Does the quadratic (in α)

$$\|v\|^2 \alpha^2 - 2(u \cdot v) \alpha + \|u\|^2 - r^2$$

have any real, non-negative roots?

• No: no intersection

• Yes: use smallest non-negative root for α

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intersection

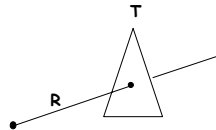
- sphere
- triangle
- box
- cylinder
- cone
- torus

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triangle intersection



- do R and T intersect?
- if so, where?

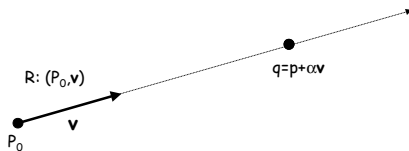
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ray: parametric form

a point q lies on $R = (p, v)$ iff $q = p + \alpha v$ for some $\alpha \geq 0$



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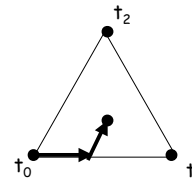
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triangle: parametric form

a point q lies on the triangle T iff $q = t_0 + \beta u + \gamma w$ where

$$u = t_1 - t_0, \quad w = t_2 - t_0, \quad \text{and } \beta \geq 0, \gamma \geq 0, \beta + \gamma \leq 1$$

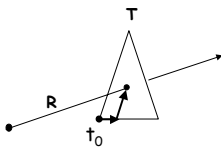


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triangle intersection



Find α, β, γ such that

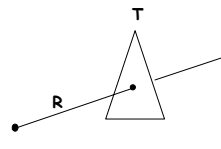
$$P_0 + \alpha v = t_0 + \beta u + \gamma w$$

$$\alpha, \beta, \gamma \geq 0$$

$$\beta + \gamma \leq 1$$

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triangle intersection - take 1



Find α, β, γ such that

$$P_0 + \alpha v = t_0 + \beta u + \gamma w$$

$$\alpha, \beta, \gamma \geq 0$$

$$\beta + \gamma \leq 1$$

System of linear equations

$$P_{0x} + \alpha v_x = t_{0x} + \beta u_x + \gamma w_x$$

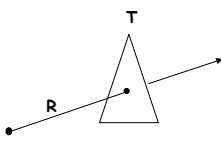
$$P_{0y} + \alpha v_y = t_{0y} + \beta u_y + \gamma w_y$$

$$P_{0z} + \alpha v_z = t_{0z} + \beta u_z + \gamma w_z$$

Invert the matrix. Yuck. Is there another way?

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triangle intersection - take 2

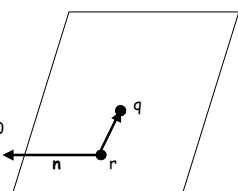


1. Find the intersection point (if any) of R and the plane containing T.
2. Find the parameterization of the intersection point in terms of β, γ .

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plane

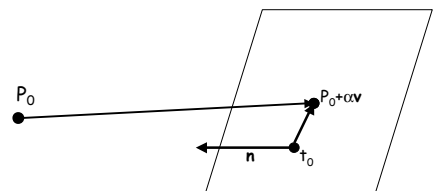
Let n be a normal to the plane.
 Let r be a point on the plane.
 Let q be an arbitrary point in space.
 Then q lies on the plane iff $n \cdot (q - r) = 0$



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plane intersection

Is there a point $P_0 + \alpha v, \alpha \geq 0$, such that $n \cdot (P_0 + \alpha v - t_0) = 0$?



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plane intersection

Is there a point $P_0 + \alpha v, \alpha \geq 0$, such that $n \cdot (P_0 + \alpha v - t_0) = 0$?

Assume there is and solve for α .

Let r be the vector from P_0 to t_0 .

Then $0 = n \cdot (P_0 + \alpha v - t_0) = n \cdot (\alpha v - r) = \alpha(n \cdot v) - n \cdot r$.

If $n \cdot v = 0$ then R is either parallel to plane or it lies in the plane.

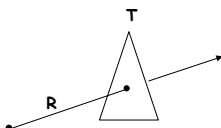
If $n \cdot v \neq 0$ then $\alpha = (n \cdot r) / (n \cdot v)$.

If $\alpha \geq 0$ then R intersects the plane at point $P_0 + \alpha v$.

If $\alpha < 0$ then R does not intersect plane.

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triangle intersection - take 2

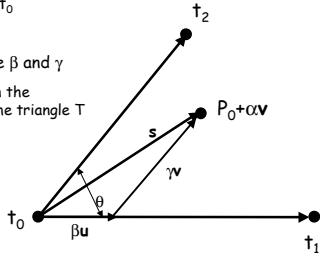


1. Find the intersection point (if any) of R and the plane containing T.
2. Find the parameterization of the intersection point in terms of β, γ .

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triangle intersection

recall $u=t_1-t_0$ and $w=t_2-t_0$
 let $s = P_0 + \alpha v - t_0$.
 then $s = \beta u + \gamma w$ for some β and γ
 if $\beta, \gamma \geq 0$ and $\beta + \gamma \leq 1$ then the intersection point lies in the triangle T



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triangle intersection

To find β and γ solve system of equations:
 $\beta(u \cdot w) + \gamma(w \cdot w) = s \cdot w$
 $\beta(u \cdot u) + \gamma(w \cdot u) = s \cdot u$

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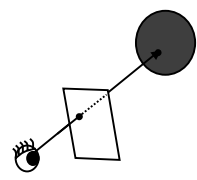
intersection

- triangle
- sphere
- box
- cylinder
- cone
- torus

} Etc.

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ray casting



- cast ray through pixel into scene
- find intersection point (if any) that is closest to eye
- **compute luminance at intersection**

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luminance

the luminance of a point on a surface depends on

- lights in scene
- material properties of surface
- geometry of scene

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luminance

for each channel we'll **approximate** the luminance at the intersection point as the sum of five terms

- emission
- ambient
- diffuse reflection
- specular reflection
- transmission (refraction)

We'll describe the terms for the red channel.

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luminance

for each channel we'll approximate the luminance at the intersection point as the sum of five terms

- **emission**
- ambient
- diffuse reflection
- specular reflection
- transmission

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red emission term

mer the material red emission

NOTE: terms shown in red are input parameters

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luminance

for each channel we'll approximate the luminance at the intersection point as the sum of five terms

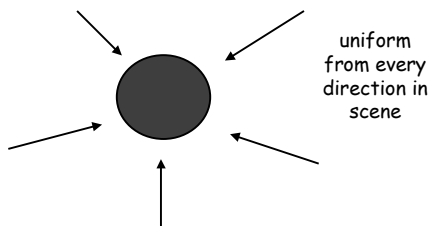
- emission
- **ambient**
- diffuse reflection
- specular reflection
- transmission

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ambient light



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red ambient term

the red ambient term is $ar * mar$ where

- *ar* is the red intensity of the ambient light
- *mar* is the response of the surface to red ambient light

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luminance

for each channel we'll approximate the luminance at the intersection point as the sum of five terms

- emission
- ambient
- **diffuse reflection**
- specular reflection
- transmission

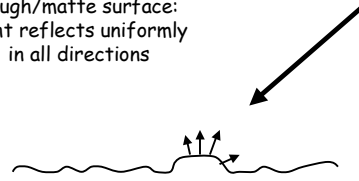
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diffuse reflections

rough/matte surface:
light reflects uniformly
in all directions



diffuse reflections provide the
surface "color"

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red diffuse term

the red diffuse reflection term is $(1 - k_{trans}) \sum R_{L,D}$
where

- the summation is taken over all lights L
- $R_{L,D}$ is the intensity of the red, diffuse reflection of light L at the intersection point
- k_{trans} is the transmission coefficient of the surface

Note: terms in green are explained in
subsequent slides

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$R_{L,D}$

depends on

- type of light
- geometry of scene
- material properties of surface

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types of lights

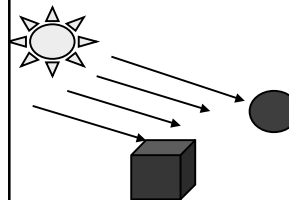
- directional light
- point light
- spot light

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directional light



- light positioned at "infinity"; intensity and incident angle are constant for all points in scene
- specification
 - direction
 - red, green, and blue intensity

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$R_{L,D}$ for directional light L

$R_{L,D} = 0$ if L is occluded
 $R_{L,D} = mdr \cdot lr \cdot \max(0, (\mathbf{n} \cdot -\mathbf{ld}))$
 otherwise

- mdr is the diffuse response of the surface material to red light
- lr is the red intensity of light L
- \mathbf{n} is the unit normal of the surface at the point of intersection
- \mathbf{ld} is a unit vector in direction the light falls

Note: you need to compute terms in blue

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normal

outward facing normal

reflection decreases as θ increases. if $|\theta| > 90^\circ$ the light falls on the back of the surface and is not reflected

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occlusion (shadows)

outward facing normal

intersection point P

light L is occluded if the ray $R=(P,-\mathbf{ld})$ intersects some object in the scene

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occlusion: implementation

offset R' slightly so it doesn't intersect at P

intersection point P

L is occluded if the ray $(P,-\mathbf{ld})$ intersects some object in the scene

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point light

- light emanates uniformly in all directions
- specification
 - location in world coordinates
 - red, green, and blue intensity
 - how the light drops off with distance

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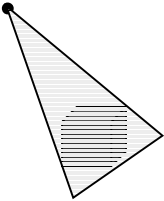
$R_{L,D}$ for point light L

$R_{L,D} = 0$ if L is occluded
 $R_{L,D} = A \cdot mdr \cdot lr \cdot \max(0, \mathbf{n} \cdot -\mathbf{ld})$
 otherwise

- mdr , lr , and \mathbf{n} are as previously defined
- \mathbf{ld} is the unit vector from the light position P_L to the intersection point
- $A = 1/(ca + la \cdot d + qa \cdot d^2)$ is the attenuation term, d is the distance between the light and the surface point

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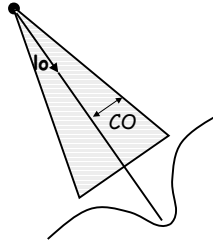
spot light



- light emanates in a cone
- specifications
 - location in world coordinates
 - red, green, and blue intensity
 - how the light drops off with distance
 - **how light drops with angle from center**

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spot light



- light emanates in a cone
- specifications
 - location in world coordinates
 - red, green, and blue intensity
 - how the light drops off with distance
 - **how light drops with angle from center**
 - light orientation vector lo
 - cutoff angle CO
 - drop off exponent DO_{62}

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$R_{L,D}$ for spot light L

$R_{L,D} = 0$ if L is occluded

$$R_{L,D} = A \cdot SP \cdot mdr \cdot lr \cdot \max(0, (n \cdot -ld))$$

where

- A, mdr, lr, and ld are as previously defined
- SP is the "spot light effect"
 - $SP=0$ if angle between lo and ld is greater than CO
 - $SP = \max(0, ld \cdot lo)^{28 \cdot DO}$ otherwise

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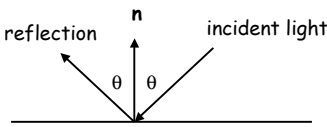
luminance

for each channel we'll approximate the luminance at the intersection point as the sum of five terms

- emission
- ambient
- diffuse reflection
- **specular reflection**
- transmission

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specular reflections



specular reflections provide highlights

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red specular

the red specular reflection term is $\sum R_{L,S}$ where

- the summation is taken over all lights L
- $R_{L,S}$ is intensity of the red, specular reflection of light L at the intersection point (details to follow shortly)

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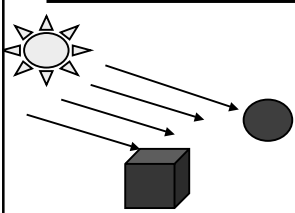
$R_{L,S}$

depends on

- type of light
- geometry of scene
- material properties of surface

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directional light



light positioned at infinity; intensity and incident angle are constant for all surface points in scene

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$R_{L,S}$ for directional light L

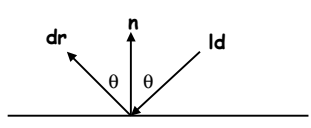
$R_{L,S} = 0$ if light is occluded

$R_{L,S} = msr \cdot lr \cdot \max(0, -v \cdot dr)^{128} \cdot k_{spec}$ otherwise

- msr is the specular response of the surface material to red light
- lr is as previously defined
- v is the direction of the incoming ray
- dr is a unit vector in the direction of reflection
- k_{spec} is the shininess constant

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vector of reflection

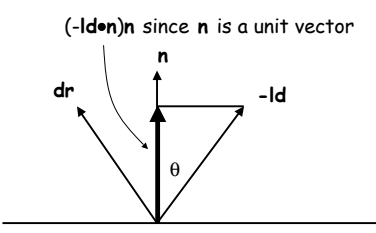


compute dr given n and ld
(all unit vectors)

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vector of reflection

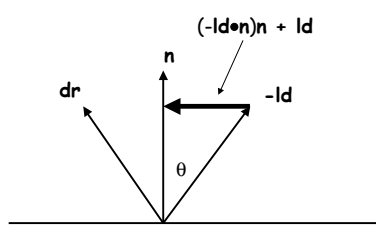
$(-ld \cdot n)n$ since n is a unit vector



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vector of reflection

$(-ld \cdot n)n + ld$



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vector of reflection

$2((-ld)n + ld)$

dr n $-ld$ θ

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vector of reflection

$dr = -ld + 2((-ld)n + ld) = ld + 2(-ld)n$

dr n $-ld$ θ

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point light

- light emanates uniformly in all directions
- specification
 - location in world coordinates
 - red, green, and blue intensity
 - how the light drops off with distance

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$R_{L,S}$ for point light L

$R_{L,S} = 0$ if light is occluded

$R_{L,S} = A \cdot msr \cdot /r \cdot \max(0, (-v \cdot dr))^{k_{spec}}$ otherwise

- all terms are as previously defined

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spot light

- light emanates in a cone
- specifications
 - location in world coordinates
 - red, green, and blue intensity
 - how the light drops off with distance
 - how light drops with angle from center

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$R_{L,S}$ for spot light L

$R_{L,S} = 0$ if L is occluded

$R_{L,S} = A \cdot SP \cdot msr \cdot /r \cdot \max(0, (-v \cdot dr))^{k_{spec}}$ otherwise

- all terms are as previously described

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luminance

for each channel we'll approximate the luminance at the intersection point as the sum of five terms

- emission
- ambient
- diffuse reflection
- specular reflection
- **transmission**

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red transmission term

the red transmission term is $\sum R_{L,T}$ where

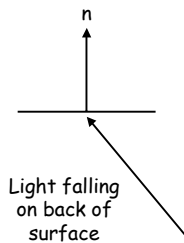
- the summation is taken over all lights L
- $R_{L,T}$ is the intensity of the red transmission of light L at the intersection point

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transmission



$$R_{L,T} = k_{\text{trans}} R_{L,D}^{\text{back}}$$

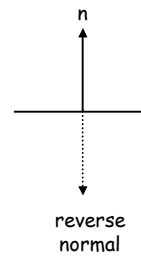
where
 • $R_{L,D}^{\text{back}}$ is the luminance of diffuse reflections on the back of the surface

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transmission computation



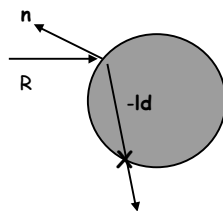
- compute $R_{L,D}$ with the reverse normal.
- scale by k_{trans}

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thick surfaces



all light falling on the back of the surface is occluded

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