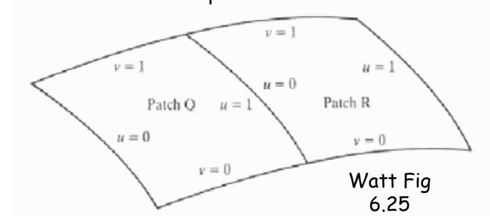
CS155: Computer Graphics

Curves & Curved Surfaces

Z Sweedyk

Bezier Surfaces

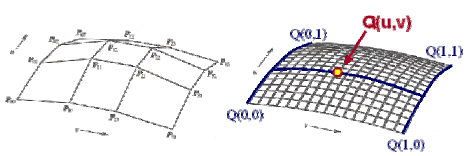
Piece together smaller patches



Watt Fig 6.25

Bezier Patch

Tensor product of Bezier curves



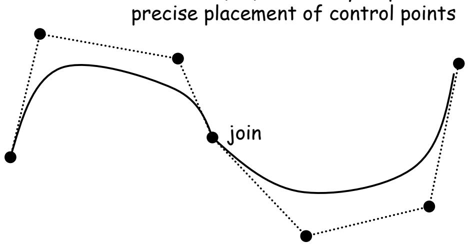
Watt Figure 6.21

Bezier Patch problems

- Boundary conditions are difficult to maintain
- Variable resolution results in cracks
- Patches are necessarily quadrilaterals

Boundary conditions

C^i & G^i (i>0) continuity require precise placement of control points



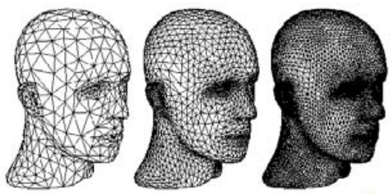
Boundary conditions

- Boundary conditions are difficult to maintain
- Solution: Enforce boundary constraints for user

This is essentially what B-splines do!

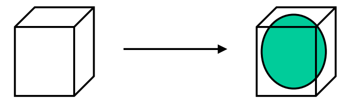
Subdivision Surfaces

Smooth surface is the limit of a subdivision process



Zorin & Schroeder SIGGRAPH 99 Course Notes

Subdivision



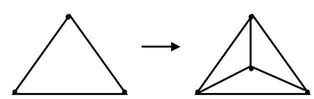
user manipulates a coarse control mesh to affect the subdivided surface

Surface Subdivision

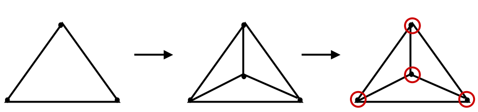
A smooth surface is computed by refining the control mesh using subdivision rules

1. Split face into multiple faces
2. Reposition vertices

Split Face



Reposition vertices

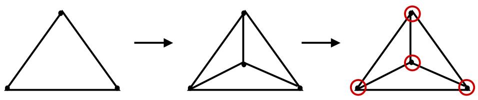


Split Face

There are two approaches to splitting a face:

- Split vertices
- Split edges

Split vertices



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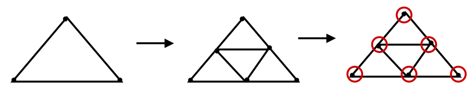
Split Edges



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Reposition vertices

- There are a variety of rules for re-positioning vertices.
- This is tricky. Subdivision process must converge to a smooth surface!
- Rules typically "average" old vertex positions.
- Problem: it's hard to create creases in subdivision surfaces.

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Geri's Game

see movie!

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Loop algorithm

- Triangle mesh subdivision
- Edge splits

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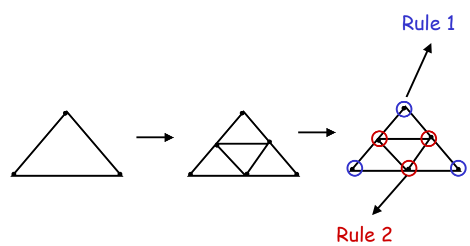
Loop Algorithm



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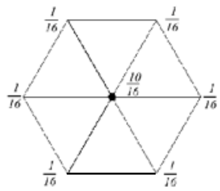
Loop Subdivision

Rule 1: Existing vertex

Case: Internal, valence 6



Note: Weights are applied to vertex positions in the old mesh.

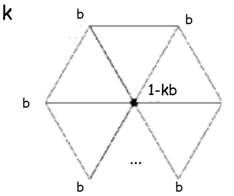
Loop Subdivision

Rule 1 : Existing Vertex

Case: Internal, valence k

K>3: b=3/8k

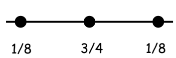
K=3: b=3/16



Loop Subdivision

Rule 1: Existing vertex

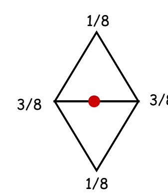
Case: Boundary



Loop Subdivision

Rule 2: New vertex

Case: Internal

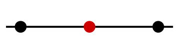


Note: Weights are applied to vertex positions in the old mesh.

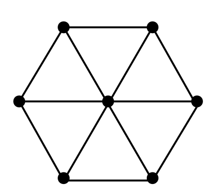
Loop Subdivision

Rule 2: New vertex

Case: Boundary

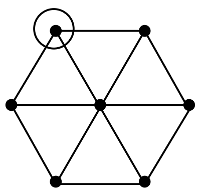


Exercise 1



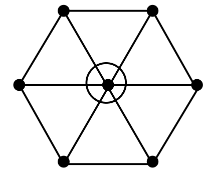
Use the Loop algorithm to split the faces of this triangle mesh.

Exercise 2



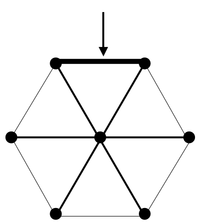
Your subdivided mesh should have a vertex corresponding to the one circled. Show how the position of this new vertex is calculated.

Exercise 3



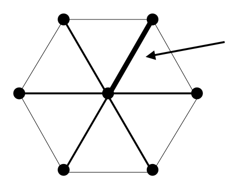
Your subdivided mesh should have a vertex corresponding to the one circled. Show how the position of this new vertex is calculated.

Exercise 4



Your subdivided mesh should have a vertex corresponding to the edge indicated. Show how the position of this new vertex is calculated.

Exercise 5



Your subdivided mesh should have a vertex corresponding to the edge indicated. Show how the position of this new vertex is calculated.

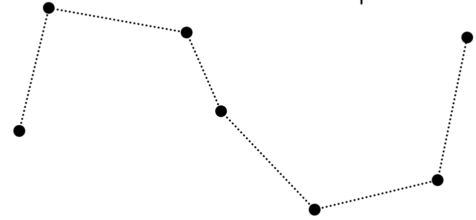
Bezier: Boundary conditions

- Boundary conditions are difficult to maintain
- Solution: Enforce boundary constraints for user

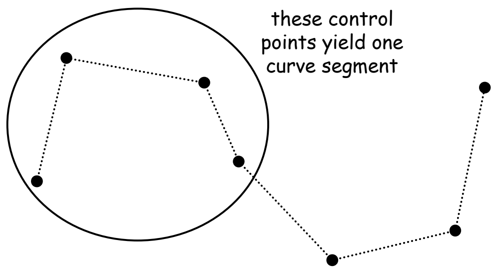
This is essentially what B-splines do!

B-Splines

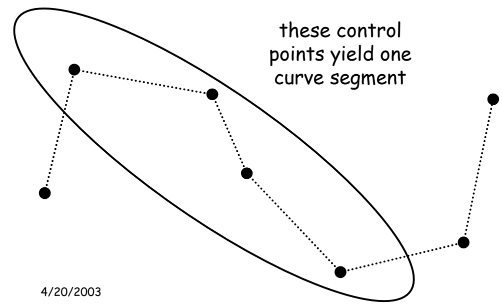
curve described by control points



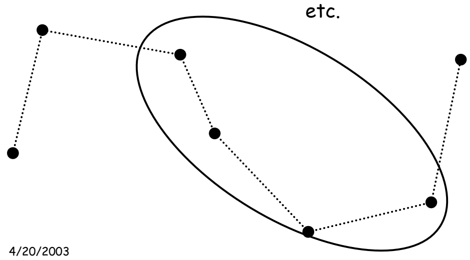
B-Splines



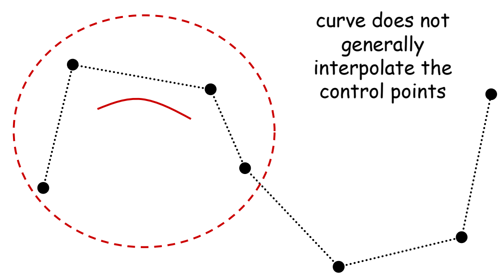
B-Splines



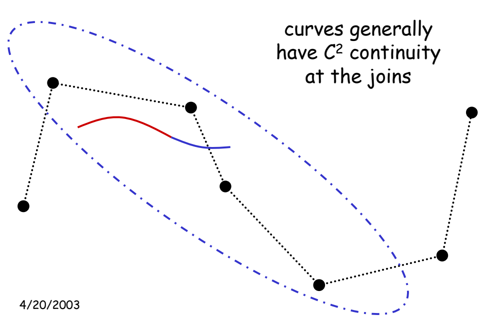
B-Splines



B-Splines



B-Splines



B-Spline

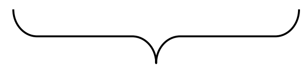
- **Uniform**
- Non-uniform

Basis Matrix: $X_i(t)$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = (1/6) \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_i \\ x_{i+1} \\ x_{i+2} \\ x_{i+3} \end{bmatrix}$$

Basis Functions: $X_i(t)$

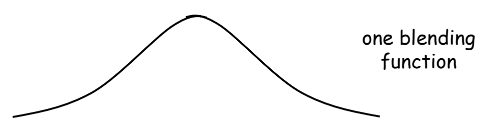
$$X(t) = [t^3 \;\; t^2 \;\; t \;\; 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_i \\ x_{i+1} \\ x_{i+2} \\ x_{i+3} \end{bmatrix}$$



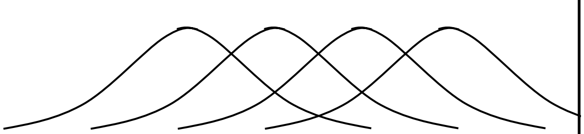
Basis Functions

- $B_0(t) = (-t^3+3t^2-3t+1)/6=(1-t)^3/6$
- $B_1(t) = (3t^3-6t^2+4)/6$
- $B_2(t) = (-3t^3+3t^2+3t+1)/6$
- $B_3(t) = u^3/6$

Blending Functions: $X_i(t)$

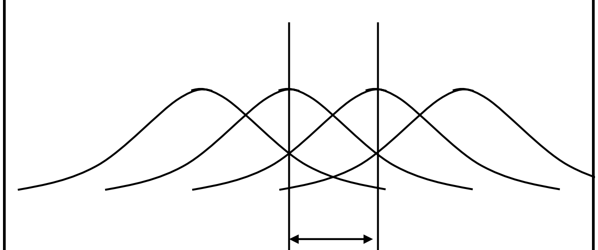


Blending Functions: $X_i(t)$



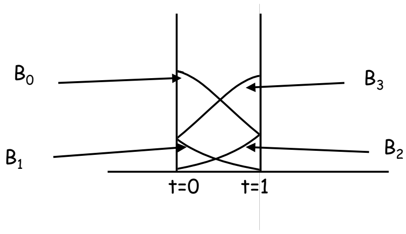
translates of blending function

Blending Functions: $X_i(t)$

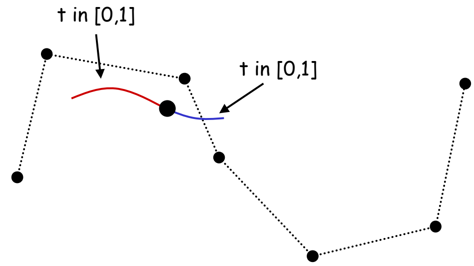


restricted to interval

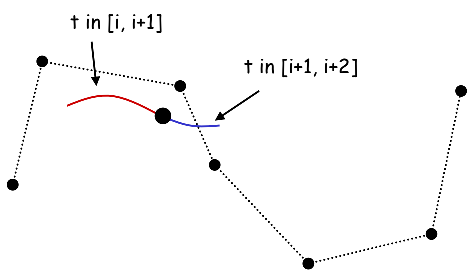
Basis Functions



Local Parameterization



Global Parameterization



Uniform B-Spline

Knots: [0, 1, 2, ..., n]

Parametric intervals of the curve segments are

[0,1], [1,2], ..., [n-1,n]

non-Uniform B-Spline

Knots: $[t_0, t_1, \ldots, t_n]$, $t_0 \le t_1 \le \ldots \le t_n$

Parametric intervals of the curve segments are

$[t_0, t_1], [t_1, t_2], \ldots, [t_{n-1}, t_n]$

non-Uniform B-Spline

next time