CS 182
Advanced Topics in Algorithms
Spring 2003
Problem Set 2b
Due Tuesday, February 4

Please remember that all problem set submissions must be typeset, preferably in LaTeX. Please keep an electronic version of your solution set for your records.

1. **[10 Points] The Arithmetic and Geometric Mean.** Let $x_1, \ldots, x_n$ be positive real numbers. The *arithmetic mean* of these numbers is defined to be $\frac{x_1+x_2+\ldots+x_n}{n}$ and the *geometric mean* is defined to be $(x_1 x_2 \cdots x_n)^{1/n}$. In this problem we show that the arithmetic mean of n numbers is at least as large as the geometric mean of those numbers.

   (a) Use induction to show that if $x_1 x_2 \cdots x_n = 1$ then $x_1 + x_2 + \ldots + x_n \geq n$. Observe that this is a special case of the statement we are trying to prove.

   (b) Use this fact to show that the arithmetic mean is at least as large as the geometric mean. (No induction required here; just a little algebra.)

2. **[15 Points] Extendible Arrays Revisited.** In class we examined extendible arrays with operations INSERT and DELETE. In particular, our rule for DELETE was that the array gets halved when the load drops below 1/4. (Recall that load is defined to be the number of elements in the array divided by the size of the array.) Now consider a different policy in which we contract the array by reducing it to 2/3 its current size when the load drops below 1/3. However, on an INSERT, we still simply double the array size when it overflows. Use the potential function

   $$\Phi(A) = |2 \times \text{num}(A) - \text{size}(A)|$$

   to show that the amortized cost of each INSERT and DELETE operation is bounded by a constant. (You can appeal to anything we did in class to save yourself some work.)

3. **[25 Points] Dictionaries.** Problem 17-2 from Cormen *et al.*, Second Edition. This problem is attached for your convenience.

4. **[25 Points] Weight-Balanced Trees!** Problem 17-3 from Cormen *et al.*, Second Edition. This problem is attached for your convenience.