Binary Relations and Graphs

Binary Relations

- A binary relation is a set of ordered-pairs, with the elements of each pair drawn from a common set.
- Example: { [1, 2], [2, 3], [1, 3] }

Binary Relations

- (A binary relation is a set of ordered-pairs, with the elements of each pair drawn from a common set, called the domain.)
- A finite set can be implemented as a list.
- An ordered-pair can be represented as a list.
- **Therefore**, a finite binary relation can be represented as a list of lists of two elements each.

Example Binary Relation Implementation

- Consider the binary relation "can be donor for" on the domain of blood types: {"A", "B", "AB", "O"}
- As a list, this relation could be represented
  [["A", "A"], ["A", "AB"],
  ["B", "B"], ["B", "AB"],
  ["AB", "AB"],
  ["O", "A"], ["O", "AB"], ["O", "B"], ["O", "O"]]

Another Binary Relation Implementation

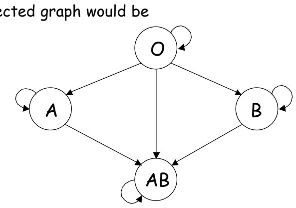
- With each first element of some pair, give the list of second elements to which the first element is related:
- [["A", ["A", "AB"]],
  ["B", ["B", "AB"],
  ["AB", ["AB"]],
  ["O", ["O", "A", "B", "AB"]]
- Compare the two representations.

Directed Graphs

- A ***Directed Graph*** is a way of presenting a binary relation:
  - The nodes (shapes) of a directed graph correspond to the elements in the domain.
  - The arcs (arrows) of a directed graph correspond to the pairs that are related.

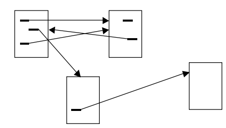
A Small Directed Graph Example

- For the binary relation can be donor for represented as a list previously
  [["A", "A"], ["A", "AB"], ["B", "B"], ["B", "AB"], ["AB", "AB"], ["O", "A"], ["O", "AB"], ["O", "B"], ["O", "O"]]
  the directed graph would be



A Large Directed Graph

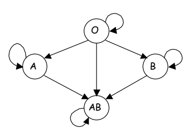
- Each page in the **world-wide web** can be considered a node.
- Each hyper-link from one page to another is an arc.



- cf. graphics.stanford.edu/papers/webviz/

Properties of Binary Relations (1 of 2)

- The previous relation example illustrates two common properties that a binary relation may have:
  - transitive property: For every x, y, z in the domain *
    if x is related to y and y is related to z,
    then x is related to z.
  - reflexive property: For every x in the domain
    x is related to x.

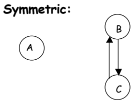


*** (Note that any of x, y, z may be equal!)**

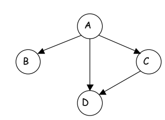
Properties of Binary Relations (2 of 2)

- The previous example has only the second of the following additional two common properties:
  - symmetric property: For every x, y in the domain
    if x is related to y then y is related to x.
  - anti-symmetric property: For every x, y in the domain
    if x is related to y and y is related to x, then
    x = y.

Symmetric:

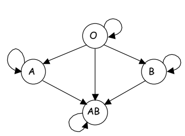


Anti-symmetric:



Partial Orders

- A relation with the reflexive, anti-symmetric, and transitive properties is called a partial order.
- Example:

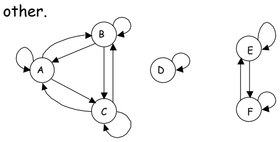


Equivalence Relations

- A relation with the reflexive, symmetric, and transitive properties is called an equivalence relation. Such a relation generalizes the notion of equality, since in this case if x is related to z and y is related to z, then x is related to y.

  In other words, if each of a set of elements is related to a common thing, the elements in the set and the common thing are all related to each other.

Equivalence:



Example of an Equivalence Relation

- Consider the relation "is a ***homophone*** of" ("sounds the same as") on a set of words, such as {"air", "ere", "heir", "buy", "by", "bye", "dew", "do", "due", "ewe", "you", "yew"}
  - Reflexive: Every x is a homophone of x.
  - Symmetric: If x is a homophone of y then y is a homophone of x.
  - Transitive: If x is a homophone of y and y is a homophone of z, then x is a homophone of z.
- Therefore this is an equivalence relation.

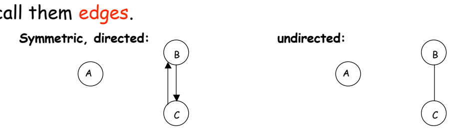
Example of not an Equivalence Relation

- Consider the relation "is a ***synonym*** for" ("means the same as")
  - Reflexive: Every x is a synomym for x.
  - Symmetric: If x is a synomym of y then y is a synomym of x.
  - Transitive: NOT: If x is a synomym of y and y is a synomym of z, then x is a synomym of z. (Example: "angry" is a synonym for "mad", and "mad" is a synonym for "insane", but "angry" is not a synonym for "insane".)
- Therefore this is a not an equivalence relation.

Undirected Graphs

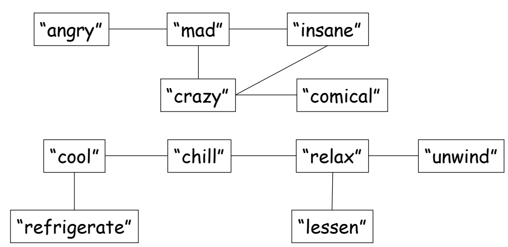
- An undirected graph is a way of presenting a symmetric binary relation:

  Since whenever x is related to y also y is related to x, we don't have to show direction with arcs. Instead of calling them arcs then, it is common to call them edges.



Undirected Graph Example

- An example of a symmetric relation and its undirected graph is "can be a synonym of":



More Information Structures?

- There is a **lot** more to be said, and our next topic in this thread will be trees.
- But for now, we will discuss some ways to work with these representations in an actual **language**.