Data Abstraction and Represenation

General Data Characterizations

- **Abstraction**: the data from a *behavioral* viewpoint (what can be done with the data)
- **Repesentation**: the data as represented in the computer (how the behaviors are implemented)
- **Presentation**: the data as presented to the user (what we see)

General Data Characterizations

Presentation

Repesentation

01101100
10110110
01011111
00001111
11101101
10101011

Example: Natural Numbers

- **Presentation: Decimal numerals:**

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, …

- **Representation: Binary:**

  0000, 0001, 0010, 0011, 0100, 0101, 0110, …

- **Abstraction**: Peano axioms

Peano Axioms (1889)

- N designates the set of natural numbers
- 0 is a particular natural number in N
- S is a function $S: N \rightarrow N$, such that:
  - For all $x \in N$ $S(x) \neq 0$.
  - For all $x, y \in N$ $S(x) = S(y)$ implies $x = y$.
- If P is any predicate, such that
  - $P(0)$
  - for all $x \in N$ $P(x)$ implies $P(S(x))$

then for all $x \in N$ $P(x)$.

Giuseppe Peano, 1858-1932

Peano vs. Decimal Presentation

- 0 is decimal is Peano 0
- 1 is $S(0)$
- 2 is $S(S(0))$
- Number n in general is $\underbrace{S(S(S(\ldots S(0)))\ldots)}_{n\ S\text{'s}}$
- $0 \neq 1$, $S(1) \neq S(2)$, etc.

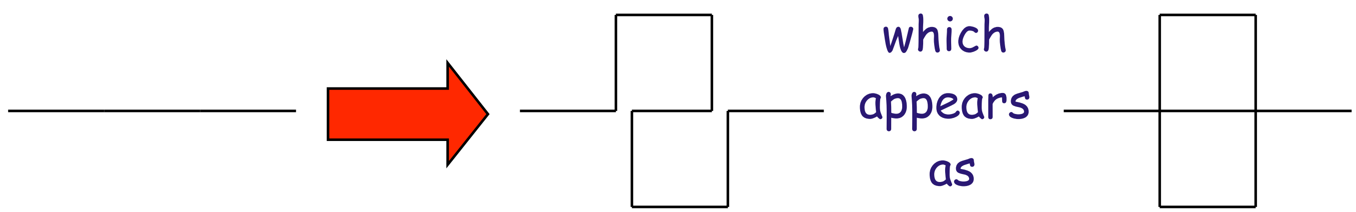
Aside: Peano's Space-Filling Curve

To fill the space of a square with a single curve:

Start with a line across the diagonal.

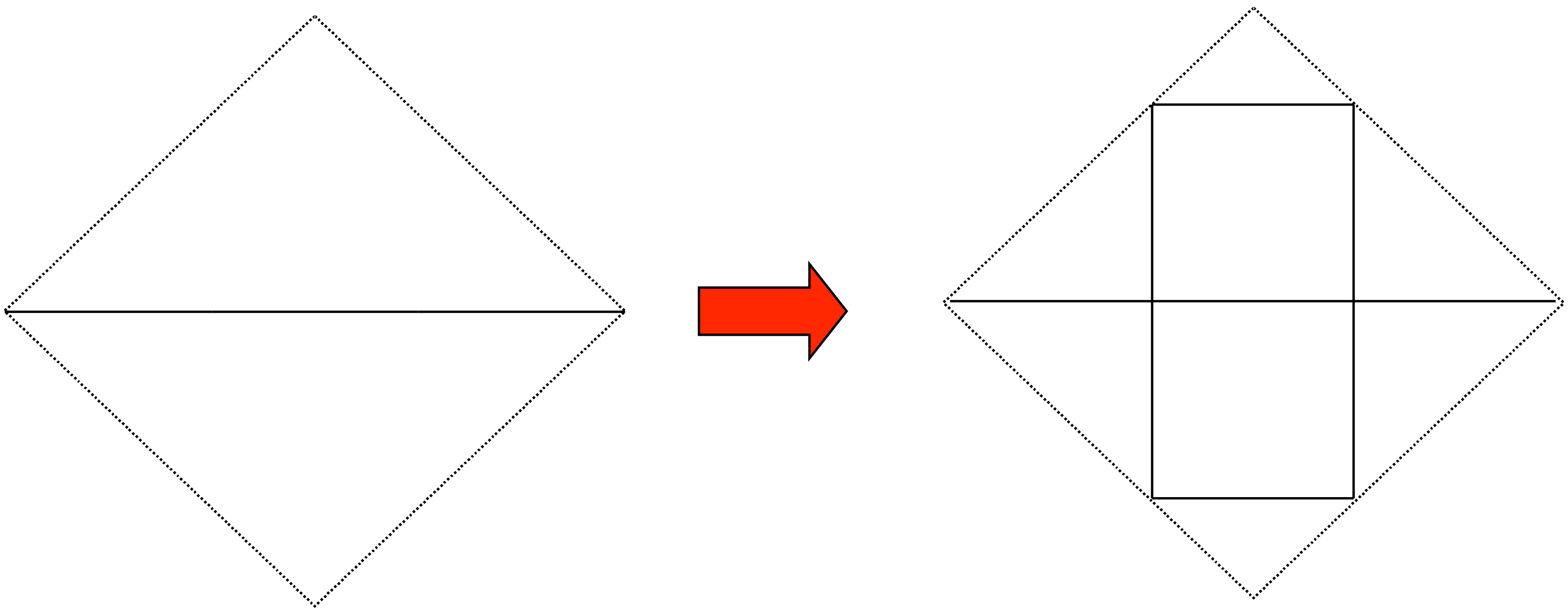
Define operation X:

Replace a line segment with a curve segment as shown:



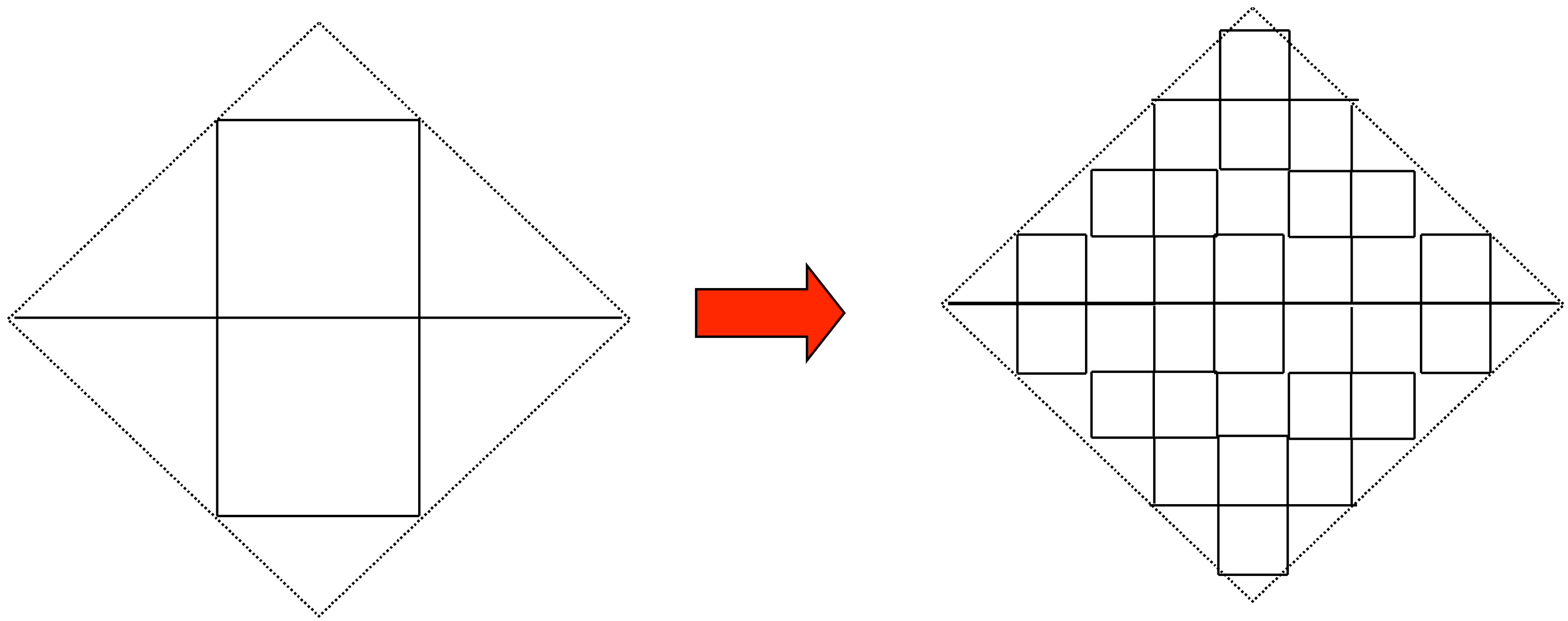
Iterate X ad infinitum

Peano's Space-Filling Curve



1st iteration

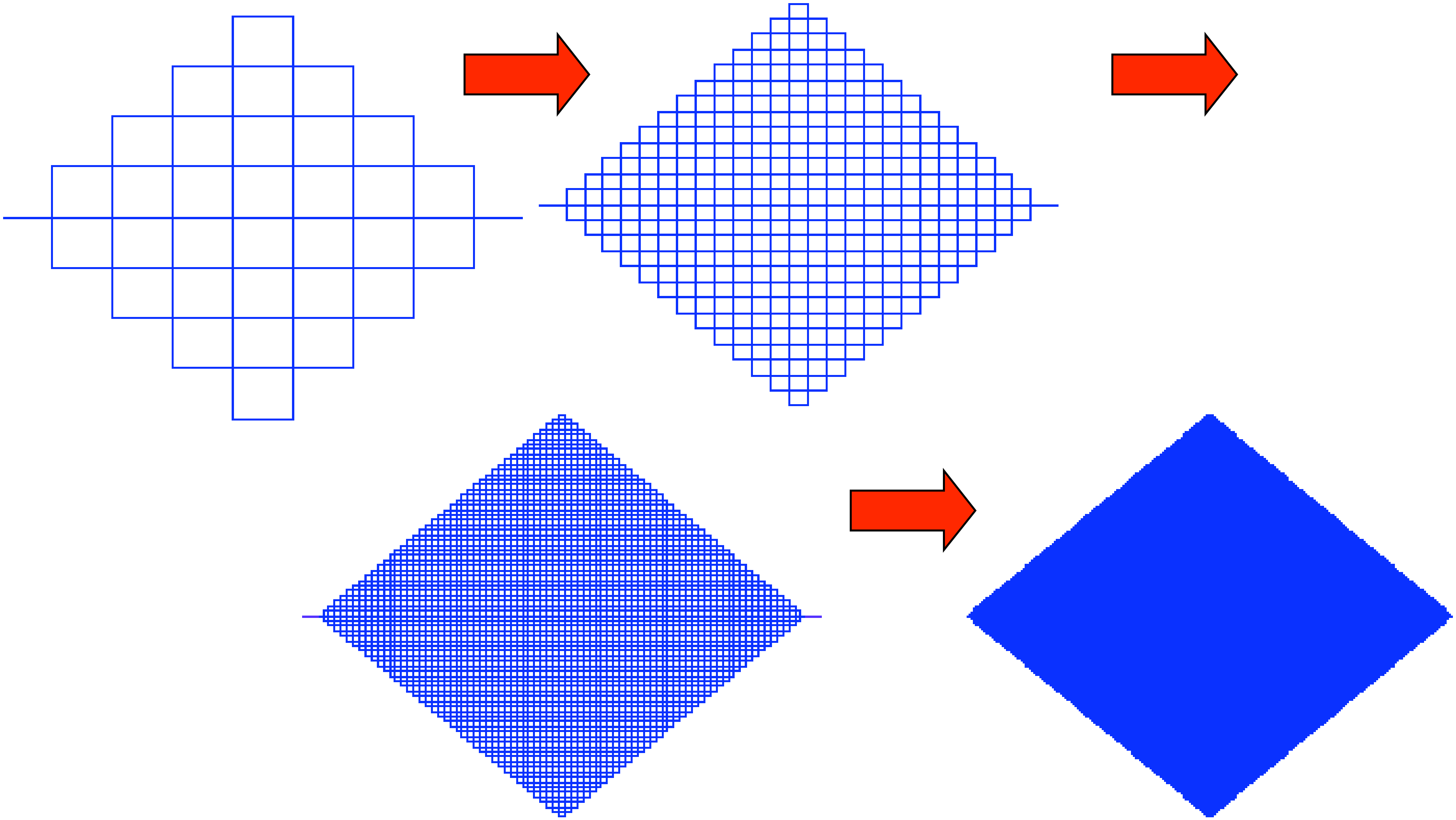
Peano's Space-Filling Curve



2nd iteration

More Peano Curve Iterations

(demo   http://library.thinkquest.org/26242/full/fm/fm25.html?tqskip1=1&tqtime=0828)

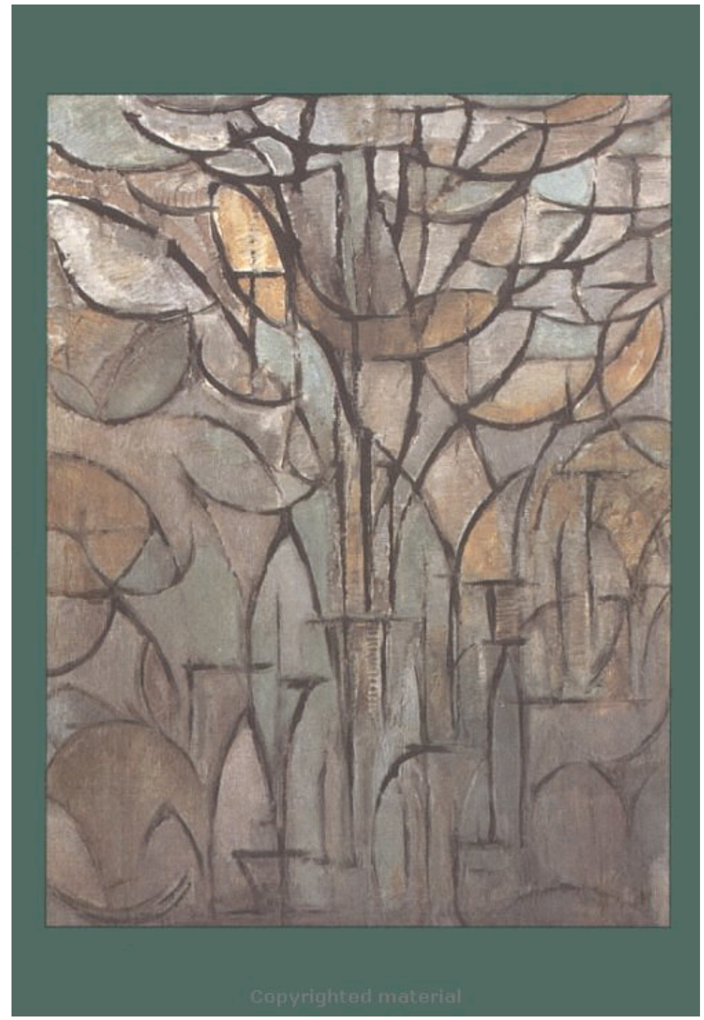


The Purpose of Abstraction in CS

- To abstract means to
  eliminate irrelevant detail.
- This is vital in presenting simple, crisp, **specifications** of what software does.
- It is also useful in **hiding detail** from those who **don't need to know** about it:
  - They can't mess with it.
  - One can change the details without changing the concept.

Abstract Art: Similar meaning, but not the same purpose

Two paintings of a tree by Mondrian.



Abstractions in Disciplines

- Chemistry is an abstraction of Physics.
- Biology is an abstraction of Chemistry.
- Genetics is an abstraction of Biology.

- Why did these abstractions evolve?

Abstraction Exercise

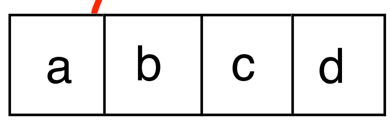
- For discussion next time:
  - Think up and describe an area outside of CS where you (or others) use abstraction.

Open-List Abstraction

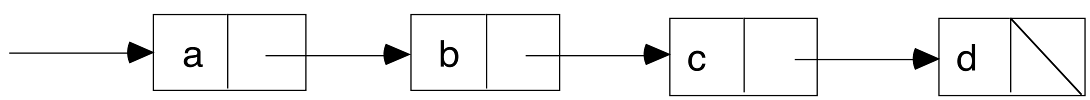
- An extension of Peano ideas
- Provides a way to create and manipulate lists in a programming language
- All definitions have a mathematical basis.
- In the text, we refer to
  - **information structures** (abstraction and presentation) vs.
  - **data structures** (implementation and representation).

Information Structures vs. Data Structures

- Information structures are an *abstraction* of data structures.
- Example: A "list" information structure, to give a few of many possible data structures:
  - could be an array



  - or could be a linked list



- Each of these is an *representation* or *implementation* of the abstraction.

Array vs. List Implementation

- Array advantages:
  - **Constant time access** to any element based on the index of the element
  - **Less storage space**, since don't store pointers

- Linked list advantages:
  - Does **not** require **contiguous memory locations** to hold array elements; uses fragmented memory more efficiently.
  - **Less expensive to insert or remove** items.

List Abstraction

- In an abstract sense, what matters most in a list is the ***order*** of the elements.
- We don't have to say how the list is represented in the machine.
- We can just agree on some ***presentation*** or ***notation*** that shows this **order**, e.g.

  [a, b, c, d]

Idea of "Structure"

- **Information** is composed of:
  - **Primitives**: *atomic* units of an agreed-upon universe, such as:
    - numbers
    - strings

    regarded as "indivisible" for the current discussion.

  - **Structures**: *collections* of information, possibly with imposed ordering information

List Structures

- Lists notation (presentation) we will often use:

  [2, 3, 5, 7]

- The notation resembles ones you've seen for sets

  {2, 3, 5, 7}

- Distinctions:
  - **Order matters** with lists; it doesn't for sets.
  - **Duplication matters** in lists; it doesn't for sets.

Equality for Lists

- Two **lists** are defined to be ***equal*** when they have the same number of elements, and their elements occur in the same order.
- Examples:

  [1, 2, 3] is equal to [1, 2, 3]

  [1, 2, 3] is not equal to [3, 1, 2]

  [1, 2, 3] is not equal to [1, 1, 2, 3]

The (one and only) Empty List

- The list with no elements
- The empty list is notated:

  [ ]

- Also called the "null list"

Lists of Various Types of Elements

- List of integers:
  [-3, -2, -1, 0, 1, 2, 3]

- List of floats:
  [3.14, 6.0238e23, -0.4567]

- List of strings:
  [“Mary”, “had”, “a”, “little”, “dog”]

Mixing types of elements

- *Can* we mix types of elements?
  Yes!

- *Should* we mix types of elements?
  Not if avoidable, but may be convenient.

Specialized Uses of Lists

- Pairs:

  [1, 2] [3, 4] [5, 6]

- Triples:

  [1, 2, 3]   [4, 5, 6]

- n-tuples:

  $[x_1, x_2, x_3, \ldots, x_n]$

  $[y_1, y_2, y_3, \ldots, y_n]$

Implementing Set Abstraction using Lists

- A set is not a list, but
- A set can be *represented* by a list:
  - simply ignore the ordering of the list, and
  - either:
    - ignore duplicates, or
    - guarantee no duplicates

called a representation *invariant*

- Ignoring vs. guaranteeing have advantages and disadvantages (why?)

Lists of Lists

- In order to keep track of, or manage, an arbitrary collection of lists, we can use lists with lists as elements
- List of pairs: [[1, 2], [3, 4], [5, 6]]
  - The ordering within each pair can be respected or not, as we desire (ordered vs. unordered pair)
- List of triples: [[1, 2, 3], [4, 5, 6]]
- List of assorted-size lists:
  [[1, 2, 3], [2, 3], [3], []]

Lists can be Nested Arbitrarily-Deeply

- List of lists of lists:

  [[[1, 2, 3], [2, 3]], [[3], []]]

- Lists of lists during "sort by repeated merging":

  [[3], [8], [5], [1], [2], [7], [6], [4]]
  [[3, 8], [1, 5], [2, 7], [4, 6]]
  [[1, 3, 5, 8], [2, 4, 6, 7]]
  [[1, 2, 3, 4, 5, 6, 7, 8]]

Length of a List

- The length, or number of elements, in a list is the number at the "top level"

$$[\ \underbrace{[\ [1, 2, 3], [2, 3]\ ]}, \underbrace{[\ [3], []\ ]}\ ]$$

has length 2

$$[\underbrace{[1, 2, 3, 4]}, \underbrace{[[1, 2], [3, 4]]}, \underbrace{[[[1, 2, 3, 4]]]}]$$

has length 3

The *member* function

- **member** tells whether a specified element is in a specified list. It returns 1 or 0 accordingly:

- member(11, [5, 7, 11, 13]) ⇨ 1
- member(12, [5, 7, 11, 13]) ⇨ 0
- member(3, [[[1, 2, 3], [2, 3]], [[3], []]]) ⇨ 0

Implementing
Other Information Structures
using Lists

Association Lists

- An association list is a list of pairs.
  [["January", 31], ["February", 28], ["March", 31], ["April", 30]]

- Typically all first elements of the pairs are of the same type, and all second elements are of the same type.

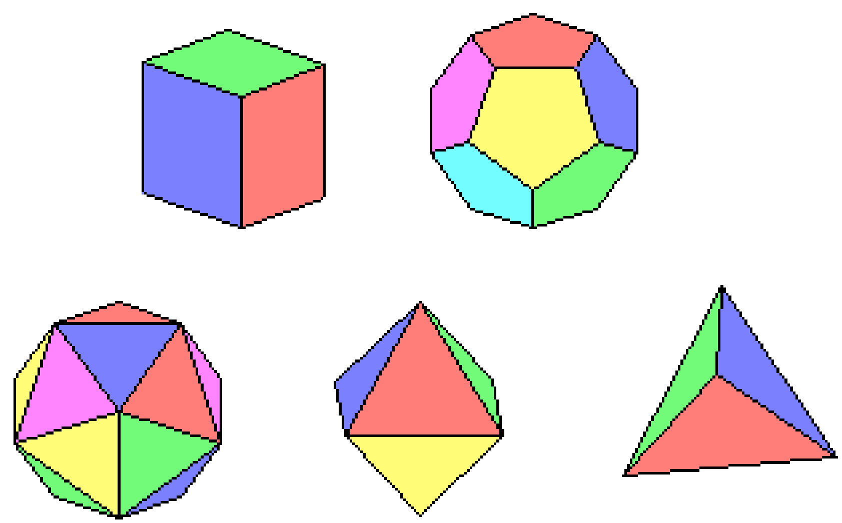
- The pairs are not necessarily of the same type as each other.

Implementing an Ordered Dictionary

- A dictionary is an abstraction associating a value with each member of a set (called the domain).
- An ordered dictionary does this while keeping the domain ordered as well.
- A (finite) ordered dictionary can be implemented as an association list.

Ordered Dictionary Example

- Implement a dictionary of regular polyhedra as an association list:
  - With each name is associated a **pair**: [*number-of-faces*, *number-of-sides-per-face*]
- [["cube", [6, 4]],
  ["dodecahedron", [12, 5]],
  ["icosahedron", [20, 3]],
  ["octahedron", [8, 3]],
  ["tetrahedron", [4, 3]]
  ]



Using a Dictionary
rex function *assoc*

- The built-in function assoc behaves as follows:
  - It has two arguments:
    - The first argument is a member of a domain, say D.
    - The second argument is an association list with domain D.
  - The result is the first pair in the association list in which the first element matches the first argument.
  - If there is no match, [ ] is returned.
    ([ ] is not a pair, so the meaning is clear.)

Example using assoc:

```
// Definition of polyhedra

polyhedra =
    [ ["cube",          [6, 4]],
      ["dodecahedron", [12, 5]],
      ["icosahedron",  [20, 3]],
      ["octahedron",   [8, 3]],
      ["tetrahedron",  [4, 3]]
   ];

// Expression to be evaluated

assoc("octahedron", polyhedra);

// Expected result:

["octahedron", [8, 3]]
```

Using the rex builtin 2-ary **test** function

```
//   Expression to be tested              Desired result
test(assoc("octahedron", polyhedra), ["octahedron",   [8, 3]]);
```

Sample session:

```
turing ~:1> rex polyhedra.rex
ok: assoc("octahedron", [[cube, [6, 4]], [dodecahedron, [12, 5]],
 [icosahedron, [20, 3]], [octahedron, [8, 3]],
 [tetrahedron, [4, 3]]]) ==> [octahedron, [8, 3]]
polyhedra.rex loaded
1 rex > ^D
turing ~:1>
```

Control-D means end-of-input

If there were an error, it would be indicated with "bad" instead of "ok" and the error count would be reported at the end.