1. [20 points]

The rex language includes a built-in function \texttt{count} defined as follows:

\begin{verbatim}
count(P, L)
\end{verbatim}

where \( P \) is a predicate and \( L \) is a list, returns the number of items in \( L \) that satisfy \( P \). For example,

\begin{verbatim}
count(odd, [1, 2, 3, 4, 5]) ==> 3
\end{verbatim}

\textbf{a.} [3/20 points] Suppose that \texttt{count} were not available, but functions \texttt{keep} and \texttt{length} were. Define \texttt{count} in terms of these other functions.
Solution: \[ \text{count}(P, L) = \text{length}(\text{keep}(P, L)); \]

b. [5/20 points] Suppose that no high-level functions were available. Define \text{count} using recursion.

Bonus: You get 5 extra points if you give a solution that is tail-recursive.

Solution:

// Non-tail-recursive version
\[
\text{count}(P, []) \Rightarrow 0;
\]
\[
\text{count}(P, [A \mid L]) \Rightarrow P(A) + \text{count}(P, L);
\]

// Tail-recursive version, where Acc is an accumulator:
\[
\text{count}(P, L) = \text{count}(P, L, 0);
\]
\[
\text{count}(P, [], \text{Acc}) \Rightarrow \text{Acc};
\]
\[
\text{count}(P, [A \mid L], \text{Acc}) \Rightarrow \text{count}(P, L, P(A) + \text{Acc});
\]

c. [5/20 points] The higher-order function \text{makeCounter} has a predicate as its single argument. It returns a function that also has one argument, a list. The latter function, when applied to a list, returns the number of elements for which \( P \) is true. For example, if we have the definition

\[
\text{counter} = \text{makeCounter}(\text{odd});
\]

then a use of this counter might be

\[
\text{counter}([1, 2, 3, 4, 5]) \Rightarrow 3
\]

Construct a rex definition of \text{makeCounter} (you may use your work from previous parts):

Solution:

\[
\text{makeCounter}(P) = ((L) \Rightarrow \text{count}(P, L));
\]
d. [7/20 points] We now want to redefine `count` to apply to *unlabeled trees* represented as lists. We want to count the number of leaves in the tree satisfying the predicate. For example:

```
count(odd, [[0, 1], [2, 3], [4, [5, 6, 7]]]) ==> 4
```

**Define the tree version of count.** You may use recursion or higher order functions, as you see fit. (The predicate `atomic` differentiates non-lists from lists.)

**Solution:**

```
count(P, T) => atomic(T) ? P(T);
count(P, T) => reduce(+, 0, map(makeCounter(P), T));
```

2. [20 points]

We wish to implement `count` mentioned in problem 1, parts a and b for OpenLists in Java.

a. [5/20 points] **Give the two main reasons why the following implementation is undesirable.**

```java
static int count(Function1 P, OpenList L) {
    int result = 0;
    for( int i = 0; i < L.length(); i++ ) {
        if( ((Boolean)P.apply(L.nth(i))).booleanValue() ) {
            result++;
        }
    }
    return result;
}
```

**Solution:**

The implementation is highly inefficient:

1. The length of list `L` is repeatedly re-evaluated.
2. The repeated use of nth(i) unnecessarily traverses the list L on each iteration.
   Consequently, the time taken to execute this function is proportional to the square of the length of the list, when simply proportional to the length of the list is adequate.

b. [5/20 points] Give an implementation that corrects the undesirable aspects in the preceding implementation.

   Solution:

   ```java
   static int count(Function1 P, OpenList L) {
       int result = 0;
       for( ; L.nonEmpty(); L = L.rest() ) {
           if((Boolean)P.apply(L.first())).booleanValue() ) {
               result++;
           }
       }
       return result;
   }
   
   Both the cast to Boolean and the extraction booleanValue() are needed. Function1 returns an Object, and since that function serves as a predicate, we package the return value as a Boolean. But we cannot test a Boolean, only a boolean, which is why we have the booleanValue() call.
   
   c. [10/20 points] Modify your implementation in part b. so that it works for the tree case, as in problem 1d.

   Solution:

   ```java
   static int count(Function1 P, Object T) {
       if( T instanceof OpenList ) {
           OpenList L = (OpenList)T;
           int result = 0;
           for( ; L.nonEmpty(); L = L.rest() ) {
               result += count(P, L.first());
           }
           return result;
       }
       else
   ```
We changed the original `OpenList` argument to an `Object` argument, then test inside whether it is either an `OpenList`, in which case it is not a leaf, or anything else, which is a leaf.

3. [20 points]

Recall that the Java interface `Iterator` has the following methods:

```java
Object next();

boolean hasNext();
```

(It also has method `remove()`, but ignore that for this problem.)

**Code a class `OpenListIterator` that implements `Iterator` for our `OpenList` class.** An object of this class will return the elements of the list, beginning with the first, until there is none left. This implementation will thus allow us to apply any code using `Iterator` to `OpenLists`. The class’s constructor will have as its only argument the `OpenList` over which iteration is to occur.

**Solution:**

```java
class OpenListIterator implements java.util.Iterator
{
    OpenList remainder;

    public OpenListIterator(OpenList theList)
    {
        remainder = theList;
    }

    public boolean hasNext()
    {
        return remainder.nonEmpty();
    }

    public Object next()
    {
        Object result = remainder.first();  // get the next value
        remainder = remainder.rest();       // modify the remainder
        return result;                      // return the next value
    }
}
```
4. [10 points]

Consider the following Java class definition in the context of closed lists:

```java
Class Cell
{
    Object data;
    Cell next;

    Cell(Object data, Cell next)
    {
        this.data = data;
        this.next = next;
    }
}
```

Suppose that variables `ap` and `bp` have been initialized as if by the following statements:

```java
Cell ap = new Cell("a", new Cell("c", null));
Cell bp = new Cell("b", null);
```

Give the sequence of assignment statements that will insert the cell containing “b” into the list destructively, between the existing cells containing “a” and “c”. (Don’t create any new cells.) **Draw the corresponding picture first.**

Solution:

```java
bp.next = ap.next;    // Make bp.next point to where ap.next points.
ap.next = bp;         // Make ap.next point to where bp points.
```

Below, the dotted lines show the situation before, that get changed.

![Diagram of list structure before and after insertion](image-url)
5. [10 points]

On the two identical copies of the directed graph below, number the nodes in the left graph in an order that could occur in a depth-first search of the graph. Number the nodes in the right graph in an order that could occur in a breadth-first search of the graph. Search from the node labeled 0 in both cases. (Nodes not reachable from that node don't get numbered.)

**Solution:**

**Depth-First:**

**Breadth-First:**

6. [20 points]

An example of exploiting inheritance concerns shapes in graphical programs. Various shapes, such as Oval, and Rectangle, are to be manipulated on the screen in a drawing program. In addition, it is possible to have a Group shape, which is a collection of constituent shapes that move as a unit. The following additional design points are of importance:

Certain methods in our application should be able to accept a shape as an argument regardless of which kind of shape it is.

Every shape has, at a minimum, an x and y position representing the upper left-hand corner of an imaginary box bounding the shape. Similarly, every shape has a width and height, defined to be the corresponding parameters of this box.
Every shape has a `move` method that will set its `x` and `y` to a designated position and a `draw` method that will cause it to be drawn on some `Graphics` object. The draw method consults the `x` and `y` of the shape to determine where to draw it.

**Show appropriate class definitions for a design that exploits inheritance.** Give key instance variables in appropriate places. Avoid replicating methods in more than one class unnecessarily. Use `OpenList` to record the shapes in a `Group`. **You do not need to code the methods**, but show headers for the methods of interest to an application program that uses your implementation.

**Solution:**

```java
// Below, note that in Shape, draw is abstract, since each type of Shape
// will generally be drawn with a different method.
// Also, move need not be abstract, since it involves setting x and y,
// which is common to all Shapes.

/**
 * Shape is an abstract base class
 */

abstract class Shape {

/**
 * horizontal position
 */

int x;

/**
 * vertical position
 */

int y;

/**
 * base class constructor
 */

Shape(int x, int y) {
    this.x = x;
    this.y = y;
}

/**
 * Draw the shape on a Graphics object.
 */

abstract void draw(Graphics graphics);
```
/**
 * Move the shape to a specific position.
 */

void move(int x, int y)
{
    this.x = x;
    this.y = y;
}

/**
 * the Rectangle shape
 */
class Rectangle extends Shape
{
    int width;
    int height;

    /**
     * Construct a rectangle.
     */
    Rectangle(int x, int y, int width, int height, Color color)
    {
        super(x, y, color);
        this.width = width;
        this.height = height;
    }

    /**
     * Draw the Rectangle at its current position.
     */
    void draw(Graphics graphics)
    {
        graphics.setColor(color);
        graphics.drawRect(x, y, width, height);
    }
}

/**
 * the Oval shape
 */
class Oval extends Shape
{
    int width;
    int height;

    /**
     * Construct an Oval.
     */
    Oval(int x, int y, int width, int height, Color color)
{  
super(x, y, color);
  this.width = width;
  this.height = height;
}

/**
 * Draw the Oval at its current position.
 */
void draw(Graphics graphics)
{
  graphics.setColor(color);
  graphics.drawOval(x, y, width, height);
}

/**
 * the Group shape
 */
class Group extends Shape
{
  OpenList contents;

  /**
   * The text content of this Text shape.
   */
  String string;

  /**
   * Construct a Group Shape.
   */
  Group(int x, int y)
  {
    super(x, y);
    contents = OpenList.nil;
  }

  /**
   * Draw the Shape at its current position.
   */
  void draw(Graphics graphics)
  {
    ... Iterate over shapes in contents, drawing them ... 
  }
}