Imperative vs. Functional Programs

Taxonomy of Programming Models

- Imperative Programming
- Declarative Programming
- Functional Programming
- Logic Programming
- Conventional Programming
- Object-Oriented Programming

Imperative Programming

- View of computation is as sequence of commands or assignments
- (vs. functional: as set of function declarations)
- Most basic operation is the assignment statement:
  \[ \text{Variable} = \text{Expression}; \]
  \[ x = x+1; \]

No "referential transparency"

- In a functional language
  \[ f(X) \times f(X) \]
  is same value as
  \[ Y = f(X), Y=Y \]
- In an imperative language, we cannot make this claim; \( f(X) \) may have "side-effects" apart from the production of a value. (In some cases, it could even modify \( X \).

Expressive Power

- Every imperative program can be expressed as an equivalent functional program
- The general idea:
  - The "state" of an imperative program consists of a set of bindings of values to variables.
  - A statement, or sequence of statements, in an imperative program can be regarded as a transformation of one state to another.
  - The transformation represented by a statement can be expressed as a function.
Example: Factorial Program

```c
int fac(int n) {
    int x, a;
    x = 1; a = 1;
    while (x < n) {
        a = a*x;
        x = x+1;
    }
    return a;
}
```

The state is the set of bindings to a, n, and x, which we'll abbreviate (a, n, x), called the state vector.

Example: (a, n, x):
(1, 4, 1) -> (1, 4, 2) -> (2, 4, 3) -> (6, 4, 4) -> (24, 4, 5)
24 is returned as fac(4)

Expressing Imperative Programs Functionally (1 of 4)

- Think of the program as represented by its flowchart.

```c
int fac(int n) {
    int x, a;
    x = 1; a = 1;
    while (x <= n) {
        a = a*x;
        x = x+1;
    }
    return a;
}
```

Label each arc with the name of a function having the state vector as an argument, except for the input arc, which gets the input variables as an argument, and the output arc, which need not be labeled.

Interpretation of the functions thus introduced:
- Given the argument values as the state, the function produces the value that the program would eventually produce if it were started in that state at the indicated arc.

Define the functions according to the state transformations in boxes.

Simplifying Using Substitution

```c
f_{1} = f_{1}(1, n, 1); 
\text{ This is called "McCarthy's transformation principle in the text. }
```
**Try this one**

```c
int fib(int n) {
    int x, a, b;
    x = 1; a = 1; b = 0;
    while (x <= n) {
        int temp = a + b;
        b = a;
        a = temp;
        x = x + 1;
    }
    return a;
}
```

**Recursion -> Iteration?**

- Is McCarthy's transformation invertible?
  - In some cases, it is possible to go from recursion to iteration, if the program is tail-recursive.
  - In general, it is not possible to transform an arbitrary recursive program to iteration, except in a fairly contrived way:
    - We can always implement recursion using imperative programming and a stack
    - In some sense, this suggests that recursive programming is strictly more expressively-powerful than iterative programming.

**John McCarthy**

A pioneer in artificial intelligence, McCarthy invented LISP, the preeminent AI programming language, and first proposed general-purpose time sharing of computers. Ph.D. Princeton, 1951. Distinctions: NAS, NAI.

[Link to McCarthy's original paper giving the transformation (IFIP 62).]

**Funky Faktorial?**

```c
fac(n) = f_1(1, n, 1);

f_1(a, n, x) = x <= n ? f_1(a*x, n, x+1) : a;
```

Compare to everyone's favorite:

```c
fac(n) = n <= 1 ? 1 : n*fac(n-1);
```

**Tail Recursion (review)**

Functions produced by McCarthy's transformation are all "tail-recursive", meaning that the result of the function can be wholly delegated to some other defined function call.

```c
fac(n) = f_1(1, n, 1);

f_1(a, n, x) = x <= n ? f_1(a*x, n, x+1) : a;

fac(n) = n <= 1 ? 1 : n*fac(n-1);
```

```
"tail-recursive"
```

**Tail Recursion**

There is no "messy cleanup" after the inner function is called.

```c
fac(n) = f_1(1, n, 1);

f_1(a, n, x) = x <= n ? f_1(a*x, n, x+1) : a;

fac(n) = n <= 1 ? 1 : n*fac(n-1);
```

```
tail-recursive
```

**Good Housekeeping Seal of Approval**
Tail Recursion

\[
\text{fac}(n) = \begin{cases} 
1 & \text{if } n \leq 1 \\
\text{fac}(n-1) & \text{otherwise}
\end{cases}
\]

\[
f_1(a, n, x) = \begin{cases} 
1 & \text{if } x \leq n \\
f_1(a \times x, n, x+1) & \text{otherwise}
\end{cases}
\]

Tail recursive

Non-tail recursive

Which should I use?

- Don’t lose sleep over whether to tail-recurse, unless
  - you are processing large data objects and memory is a premium, or
  - it is much easier to compute without it, or
  - it’s stated on the exam that you should.
- The compiler must also optimize tail-recursion for this to be effective (currently rexx doesn’t).
- In development, it might be wise to provide the clearest expression of the function first, then later replace it with a tail-recursive version.

Naïve Reverse

The valid rule set:

\[
\text{reverse}(\text{[]}) \Rightarrow \text{[]}
\]

\[
\text{reverse}([E | L]) \Rightarrow \text{append}(\text{reverse}(L), [E])
\]

is called naïve reverse:

- It’s the first reverse coded by the inexperienced.
- It’s not tail recursive.
- It’s slow: takes an extra factor of length(L) steps to evaluate.

Accumulators (review)

- Certain arguments of functions, particularly tail-recursive ones, are often designated as “accumulators”, e.g.

\[
f_1(a, n, x) = \begin{cases} 
x & \text{if } x \leq n \\
1 & \text{otherwise}
\end{cases}
\]

- The idea is that this argument value "accumulates" until the function is ready to return the answer without recursing.

Accumulators for List Processing

- Consider a definition of reverse:

\[
\text{reverse}(L) = \text{reverse}(L, [\text{ ]})
\]

\[
\text{reverse}([\text{ ]}, A) \Rightarrow A
\]

\[
\text{reverse}([E | L], A) \Rightarrow \text{reverse}(L, [E | A])
\]

- Which argument is an accumulator?
- Is this reverse tail-recursive?
### Accumulators for List Processing

<table>
<thead>
<tr>
<th>Accumulator Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>reverse(L) = reverse(L, [ ]);</td>
<td>initial accumulation</td>
</tr>
<tr>
<td>reverse([], A) =&gt; A;</td>
<td>final accumulation</td>
</tr>
<tr>
<td>reverse([E</td>
<td>L], A) =&gt; reverse(L, [E</td>
</tr>
</tbody>
</table>

### Accumulators and Auxiliaries

- Note that when an accumulator is used, it is often in an auxiliary function, rather than the main interface function for the user.

- It is bad style to burden the user with the need to know added arguments, such as initial accumulations.