Searching Trees & Graphs

Tree-Dichotomy for Recursion

- We often want a different induction/recursion **dichotomy** for trees than for lists (empty vs. non-empty):
  - The tree is a **single node** (and is thus a **leaf**).
  - The tree is a **node with offspring** (and is thus **not a leaf**).

Model Independence

- We can abstract away the specific representation being used:
  - **isLeaf**(T) 1 when T is leaf, 0 otherwise.
  - **offspring**(T) the list of offspring of a non-leaf, undefined otherwise
- The implementation depends on which tree model we are using.

Example Implementations

- Labeled-Tree Implementation:
  - **makeTree**(Root, ListOfSubTrees) = [Root | ListOfSubTrees];
  - **isLeaf**(T) = rest(T) == [ ];
  - **getOffspring**(T) = rest(T);
  - **getRoot(T)** = first(T);

Recursion on Trees

- Basis: What happens on a single leaf.
- Induction step: What happens on a non-leaf.

Example: Height of a Tree

- The height of a tree is the length of the longest path from the root.
  - height(T) => isLeaf(T) ? 0;
  - height(T) => 1 + sum(map(height, getOffspring(T));
- Let recursion do the work for you.

Searching a Labeled Tree

- Suppose we want to find **all** nodes with labels having a property P.

Searching a Labeled Tree

- findInTree(P, T) =

  Root = getRoot(T),

  foundInRest =
  mappend((S)=>findInTree(P, S), getOffspring(T)),

  P(Root) ? [Root | foundInRest] : foundInRest;

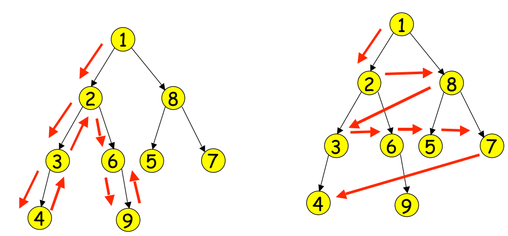
Depth-First Search

- The preceding expresses only one form of search:

  depth-first search

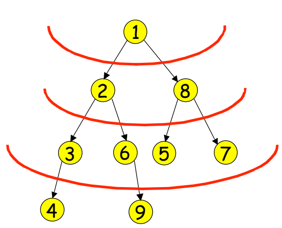
  The pattern is to search "deeper" before "broader".

Depth- vs. Breadth- First



Example: Find evens

Breadth-First: Wavefront Analogy



Advantage of Breadth-First

Breadth-First
Searching a Labeled Tree

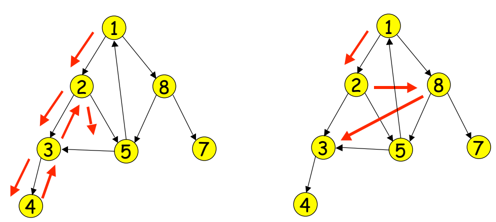
- Easier if we generalize to search a forest:
- findInTreeBF(P, T) = findInForest(P, [T]);
- findInForest(P, [ ]) => [ ];

  forest of one tree
- findInForest(P, [Tree | Trees]) =>
  Root = root(Tree),
  foundInRest = findInForest(P, append(Trees, getOffspring(Tree))),
  P(Root) ? [Root | foundInRest] : foundInRest;

Searching Graphs vs. Trees

- Basic ideas still apply, but
  - In graph, avoid re-searching same nodes due to fanin-in
  - In graph, avoid infinite loops.
  - How?

Depth- vs. Breadth- First in Graph



Example: Find evens

Searching Without Recursion

- Depth-First: Use Stack
- Breadth-First: Use Queue
- Avoid Fan-in and Cycles:
  - “Mark” nodes as encountered
    - Refuse to re-search from a marked node
  - Marking can be metaphoric, e.g. by membership on a list, or
    The node itself can be marked (non-functional programming).

Searching a Maze

- A *maze* is an implicit graph
- **Nodes** are identifiable by position
- The **arcs** are implicit, determined by adjacent spatial positions.
- Marking can be done in a “parallel array”

Recovering Path

- Searching for the first element satisfying some property.
- Want not just the element, but the **path** from the starting point to the element.
- How to accomplish?

Lineage and Back Pointers

- Non-root nodes are encountered from a parent node.
- When a node is encountered, could keep a record of the parent.
- Keep a list of nodes during descent is one means.
- Setting a "back pointer" to the parent is another; could use an association list of [child, parent] for example.

Two Birds with …

- If we use back-pointers to remember parents, we don't also need marking:
  - A node with a known parent is implicitly **marked**.
  - A node without a known parent is **not marked**.

Non-functional Back-Pointers

- When not restricted to functional programming techniques only, can use destructive setting of back-pointers for greater efficiency.