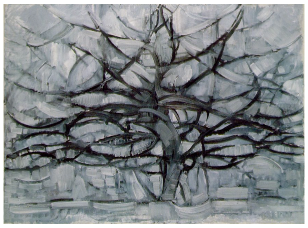
Trees and their Implementation as Lists

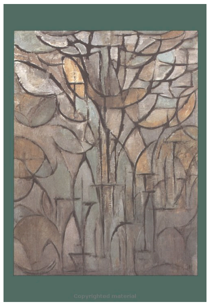
The Tree is a Pervasive Information Structure

- Files & Directories
- Family Trees
- Management Hierarchies
- Decision Trees
- Image Trees

- In the current discussion:
  - Trees are the **abstraction**
  - Lists will be the **implementation**

Maybe Mondrian Knew





Files and Directory Trees

/ ← root directory

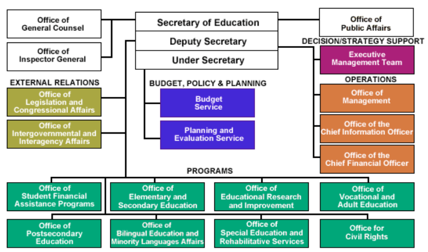
```
/
    /dev
        /dev/console
        /dev/dsk
            /dev/dsk/dsk01
            /dev/dsk/dsk02
    /etc
        /etc/mail
    /usr
        /usr/bin/emacs
        /usr/bin/ls
        /usr/bin/more
```

Family Trees

```
Joseph Patrick Kennedy* - (m. Rose Elizabeth Fitzgerald*)
 ________________________|
|
|
|-Joseph Patrick Kennedy Jr.*
|
|-Rosemary Kennedy
|
|-Kathleen Kennedy* (m. William John Robert Cavendish)
|
|-Eunice Mary Kennedy (m. Robert Sargent Shriver Jr.)
|      |
|      |-Robert Sargent Shriver III
|      |-Maria Owings Shriver (m. Arnold Schwarzenegger)
|      |-Timothy Perry Shriver
|      |-Mark Kennedy Shriver
|      '-Anthony Paul Shriver
|
|-Patricia Kennedy (m. Peter Lawford*, divorced)
|      |
|      |-Christopher Kennedy Lawford
|      |-Sydney Maleia Lawford McKelvey
|      |-Victoria Francis Lawford Ponder
|      '-Robin Elizabeth Lawford
|
|-Robert Francis Kennedy* (m. Ethel Skakel)
|      |
|      |-Kathleen Hartington Kennedy
|      |-Joseph Patrick Kennedy II
|      |-Robert Francis Kennedy Jr.
|      |-David Anthony Kennedy*
|      |-Mary Courtney Kennedy
|      |-Michael LeMoyne Kennedy*
|      |-Mary Kerry Kennedy
|      |-Christopher George Kennedy
|      |-Matthew Maxwell Taylor Kennedy
|      |-Douglas Harriman Kennedy
|      '-Rory Elizabeth Katherine Kennedy

|-Jean Ann Kennedy (m. Stephen Edward Smith*)
|      |-Stephen Edward Smith Jr.
|      |-William Kennedy Smith
|      |-Amanda Mary Smith
|      '-Kym Marie Smith
|
|-Edward Moore Kennedy  (m. #1: Virginia Joan Bennet)
|      |                (m. #2: Victoria Anne Reggie)
|      |
|      |-Kara Ann Kennedy Allen
|      |-Edward Moore Kennedy Jr.
|      '-Patrick Joseph Kennedy
|
'-John Fitzgerald Kennedy* (m. Jacqueline Lee Bouvier*)
       |
       |-unnamed daughter* (still born)
       |
       |-Caroline Bouvier Kennedy (m. Edwin A. Schlossberg)
       |     |
       |     |-Rose Schlossberg
       |     |-Tatiana Schlossberg
       |     '-John Schlossberg
       |
       |-John Fitzgerald Kennedy Jr.* (m. Carolyn Bessette*)
       |
       '-Patrick Bouvier Kennedy*
```

Organization Chart
(Management Hierarchy)



Decision Trees

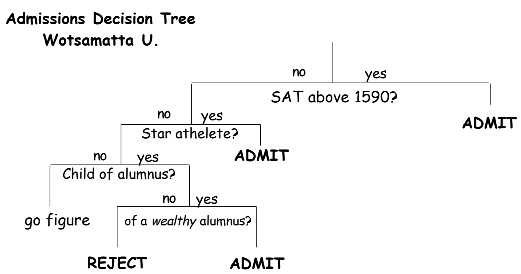
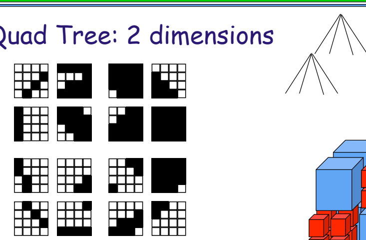
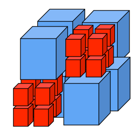


Image Trees

Quad Tree: 2 dimensions





Octree: 3 dimensions

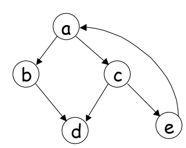
See http://www.gametutorials.com/Tutorials/OpenGL/Octree.htm

Definition of *Tree*

- There are *many* different varieties of trees.
- We can present only some of them.
- Use your knowledge of these to **generalize** to other varieties.
- We will base our definition on ***paths*** and related concepts.

Terminology for Trees: **Paths** in Directed Graphs

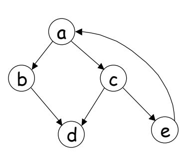
- A **path** in a graph *G* is a list of nodes n_0, n_1, …, n_k such that each successive pair (n_i, n_{i+1}) is in the corresponding binary relation.



- Some paths:
  - a, b, d
  - c, e, a
  - a, c, e, a, c, d

Cycles

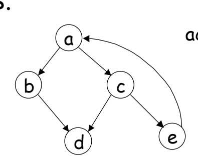
- A **cycle** is a path that starts and ends on the same node.
- Examples:
  - a, c, e, a
  - e, a, c, e, a, c, e



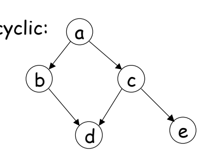
Cyclic and Acyclic

- A ***cyclic*** graph is one that has at least one cycle.
- An ***acyclic*** graph is one that has *no* cycles.

cyclic:



acyclic:

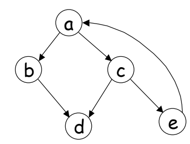


DAGs

- DAG is an acronym for "Directed Acyclic Graph"
- "DAG" is mainly used because it is more pronounceable than ADG ("Acyclic Directed Graph")

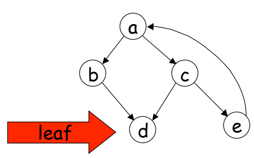
Target Set

- The ***target set*** of a node n is the set of nodes to which there is an arc from n.
  - targets(a) = {b, c}
  - targets(b) = {d}
  - targets(c) = {d, e}
  - targets(d) = { }
  - targets(e) = {a}



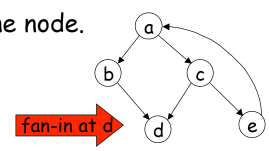
Leaves

- If a node's target set is empty, that node is called a ***leaf***.



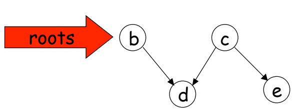
Fan-In

- A directed graph is said to ***fan-in at*** node n if the node is in the target sets of two or more different nodes.
- A directed graph "**has fan-in**" if it fans in at least one node.

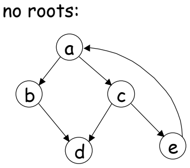


Roots

- A ***root*** of a directed graph is a node that is not in any node's target set.



no roots:

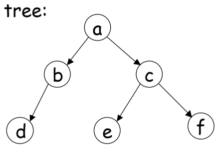


Tree at Last

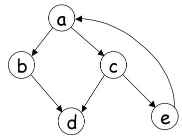
- A ***tree*** is a directed graph such that:
  - The graph is acyclic.
  - There is exactly one root.
  - It has no fan-in.

Tree vs Not

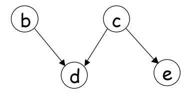
A tree:

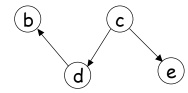


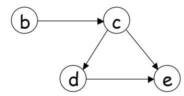
Not a tree:

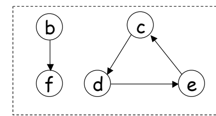


Classify these for Tree-dom

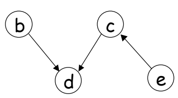


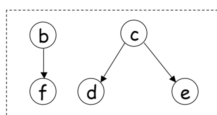


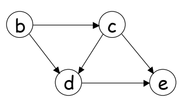


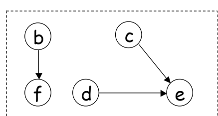


More Graphs to Classify



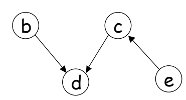


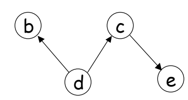




Reverse Graphs

- Some graphs that may look tree-like aren't technically trees unless we consider the **reverse graph** (one with all of the arcs of the original reversed).

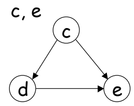




Reconvergence, an Alternative

- A **reconvergence** is a pair of *different* paths that start and end, respectively, on the *same* nodes.
- Therefore, a tree can also be characterized as a directed graph that
  - has one root
  - has no cycles
  - has no reconvergences

Reconvergence below:
c, d, e
c, e



Subsets of Three Properties

- **DAG**: acyclic, but
  - may have multiple roots,
  - may have fan-in
- **Forest**: acyclic, and no fan-in but
  - may have multiple roots
- A forest can also be characterized as a **collection of disjoint trees**.
  Each tree could be identified with its root.

Adding/Removing Arcs

- Adding arcs to a ______ that is not a tree may make it into a tree.
- Adding arcs to a ______ that is not a tree will never make it into a tree.
- Removing arcs from a ______ that is not a tree may make it into a tree.
- Removing arcs from a ______ that is not a tree will never it into a tree.

Ordered Directed Graphs

- We use the adjective ***ordered*** to indicate that the **order of targets** of a node matters.
- This property is ***implicit*** with trees much of the time.
- Because we are going to represent trees by **lists**, we can have ordering for free if we want it.

Representing/Implementing Trees as Lists

- Every tree can be represented as a list.
- Obvious:
  - Tree is a special kind of directed graph.
  - Every directed graph can be represented as a list of pairs.
- But we want a representation that makes it **clear** that we have a tree.

First Try: Target Sets

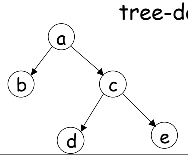
- We know that **sets** can be represented as lists.
- List the nodes of the tree.
- Associate each node with its list of targets.

Target-Lists Representation

- [[a, [b, c]],
  [b, []],
  [c, [d, e]],
  [d, [ ]],
  [e, [ ]]
  ]

nodes

targets



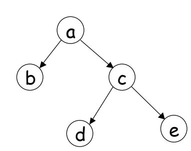
- This works for graphs in general;
  is not limited to trees.
- Doesn't directly show tree-dom.

Nested Target-Lists Representation

- Recursively, list the root, followed by the representation of each sub-tree:

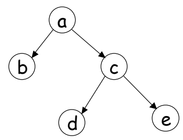
root sub-trees

- [a, ____, ____]
- [a, [b], [c, ____, ____]]
- [a, [b], [c, [d], [e]]]



Modified Nested Target-Lists Representation

- (Recall: A leaf is a node with no targets.)
- When a sub-tree is a leaf, omit the brackets around it.
  - [a, _____, _____]
  - [a, b, [c, ____, ____]]
  - [a, b, [c, d, e]]
- Less-cluttered appearance, but also less uniform processing.

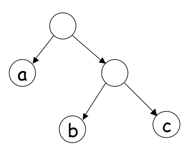


Testing leaf property

- In rex, values are either:
  - **atomic**: numbers, strings
  - **non-atomic**: lists, arrays
- atomic(X) tells whether X is atomic.

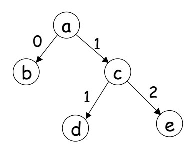
Representation of "Unlabelled" Trees

- In this model, only *leaves* have *labels*.
- A leaf is represented by its label
- A non-leaf tree is represented by a list of the representations of the targets of the root.
  - [a, _____]
  - [a, [b, c]]



Exercise

- How could you represent a tree in which both nodes and arcs have labels?



Representing Lists by Ordered Trees

- This is a kind of converse to previous discussion.
- Every list can be represented as an ordered ***binary* tree** (tree in which each node has at most two targets).
- This corresponds to a "box" storage abstraction, where the data items may themselves be lists.

Representing Lists as Trees

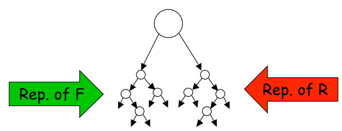
- An **atomic** item (non-list) is represented by itself.
- The **null list** is represented as a leaf [].
- A list [First | Rest] is represented by a node with two targets:
  - The **left** target is the representation of First.
  - The **right** target is the representation of Rest.
- Note that ordering of targets is essential.

Representing Lists as Trees

Atom a: a

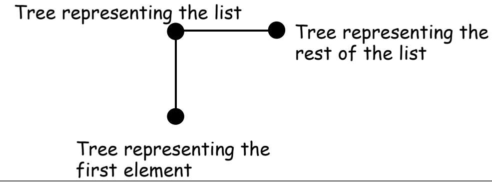
Empty list []: []

Non-empty list [F | R]:



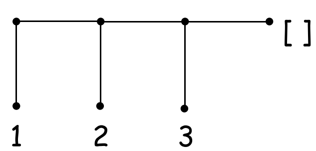
Representing Lists as Trees

- Matters are actually simpler if we rotate the tree 45°, so that "right" is horizontally right and "left" is down.



Example: Binary Tree

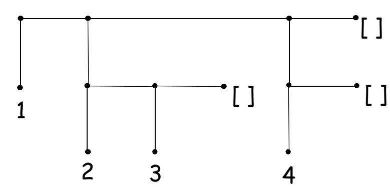
- Represent as a binary tree: [1, 2, 3]



Example: Binary Tree

- Represent as a binary tree:

  [1, [2, 3], [4]]



Corresponding Box Diagram

[1, [2, 3], [4]]