Algorithms
Computer Science 140 & Mathematics 168
Instructor: B. Thom
Fall 2004
Homework 7a
Due on Friday, 10/15/04 (midnight)

1. **[30 Points] Detecting Cycles!**

   (a) Consider an *undirected graph* with V vertices and E edges. *The graph may be disconnected.* That is, the graph may consist of a number of separate disconnected components. Describe how depth-first search can be used to determine whether or not a graph has a cycle in linear—i.e. $O(V+E)$—time. You should assume that the graph is represented by an adjacency list. Prove your algorithmic modification is correct—why it will detect some cycle in the graph (if and only if it exists). Note: if a graph is disconnected, the algorithm should return true when at least one of its components has a cycle. Finally, analyze its runtime, demonstrating that it has the desired asymptotic behavior. (This problem might require a bit of cleverness in the algorithm's data structures in order to get the $O(V+E)$ time!)

   (b) Does your your algorithm necessarily work on *directed* graphs? Explain Briefly

   (c) Does your above algorithm work if you use breadth-first search instead of depth-first search? Explain briefly.

   (d) Show how the topological sorting algorithm can be used to determine whether a *directed* graph contains a cycle (the graph may be disconnected). The algorithm should work in $O(V+E)$ time assuming the graph is represented as an adjacency list. Again, explain your algorithmic modification, justify its correctness, and analyze its runtime. Also, explain why your algorithm how your algorithm deals with a graph that is disconnected.

2. **[30 Points] Diameters.**

   In this problem we will consider connected undirected graphs with no cycles. The graphs are simply represented with adjacency lists. The distance between two vertices in such a graph is just the *minimum* number of edges on a path between the vertices. The *diameter* of the graph is the *maximum* distance between all pairs of vertices. *Make sure that you understand this definition well before proceeding!*

   Describe an algorithm that utilizes one of the reachability algorithms discussed in class to compute the diameter of a connected undirected acyclic graph in $O(V)$ time. You should justify why your algorithm is correct and carefully analyze and explain its running time.